V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This thesis explores the applications of regression models on data from the World War II Battle of Kursk—the greatest single tank battle in history. The analysis tools used are two regression techniques, linear regression and robust LTS regression. The results and a brief interpretation of each regression model are given in the model's corresponding section. The results obtained from the statistical regression models are intended to provide insight into the Battle of Kursk, as well as into combat modeling in general.

When all the regression models are viewed together the following conclusions are reached.

- It is observed that the original Lanchester equations do not fit to the Battle of Kursk data, and therefore may not be appropriate for modeling the combat. Of the three ill-fitting Lanchester equations, the best fit is obtained by applying the linear law, which is used for modeling ancient warfare or area fire.
- The parameters derived from Bracken and Fricker's Ardennes studies do not apply to the Battle of Kursk data. This implies that there are no unique parameters that apply to all battles.
- Another interesting result with respect to Bracken's methodology and models is that, upon a closer examination of his findings, the SSR values given by multiple p and q values are in the same vicinity, as seen when plotted using a 3-D plot. This is clearly seen in Figures 79 through 82, which cover the breadth of approaches and show a variety of fits.

Figure 79 shows the SSR values plotted versus *p* and *q* parameters using Bracken's model, with the tactical parameter, for the Ardennes Campaign data when $a = 8x10^{-9}$, $b = 1x10^{-8}$ and *d*=1.25.

The SSR values do not change much in the vicinity of the best fit. Except for the spike seen on the upper far right corner, the SSR values are relatively insensitive to *p* and *q* values. Figure 80 gives a closer look at the region in which the lowest SSR value is found when *p*=1 and *q*=1. A broad range of parameters fit about the same in a valley of the surface as *p* increases and *q* decreases.

For Bracken's model, without the tactical parameter, for the Ardennes Campaign data when $a = 8x10^{-9}$, $b = 1x10^{-8}$, *p*=1.3 and *q*=0.7, the same pattern observed in Figures 79 and 80 also hold true, and this is true for other *p* and *p* analyses too.

Figures 81 and 82 show the 3-D grid plots for Bracken's model with the tactical parameter for the Battle of Kursk data when $a = 1.2x10^{-8}$, $b = 9x10^{-9}$ and *d*=0.9. Figure 82 provides a closer look at the region in which the lowest SSR value is found (i.e., when *p*=0.1 and *q*=2.0). This pattern is similar to what we have seen for Ardennes.

For Bracken's model, without the tactical parameter, for the Battle of Kursk data when $a = 1.2x10^{-8}$, $b = 9x10^{-9}$, *p*=0.3 and *q*=1.8, the same pattern observed in Figure 81 and Figure 82 also holds true. This result is again similar to what was found for the Ardennes data, suggesting that a broad range of possible models fit just about as well.

In the light of the findings stated above, it is a logical next step to have a look at the model's residual surface when *a* and *b* depend on *p* and *q*, and *d*=1. The results are given in Figures 81 through 84, where *a* and *b* parameters are chosen to minimize SSR

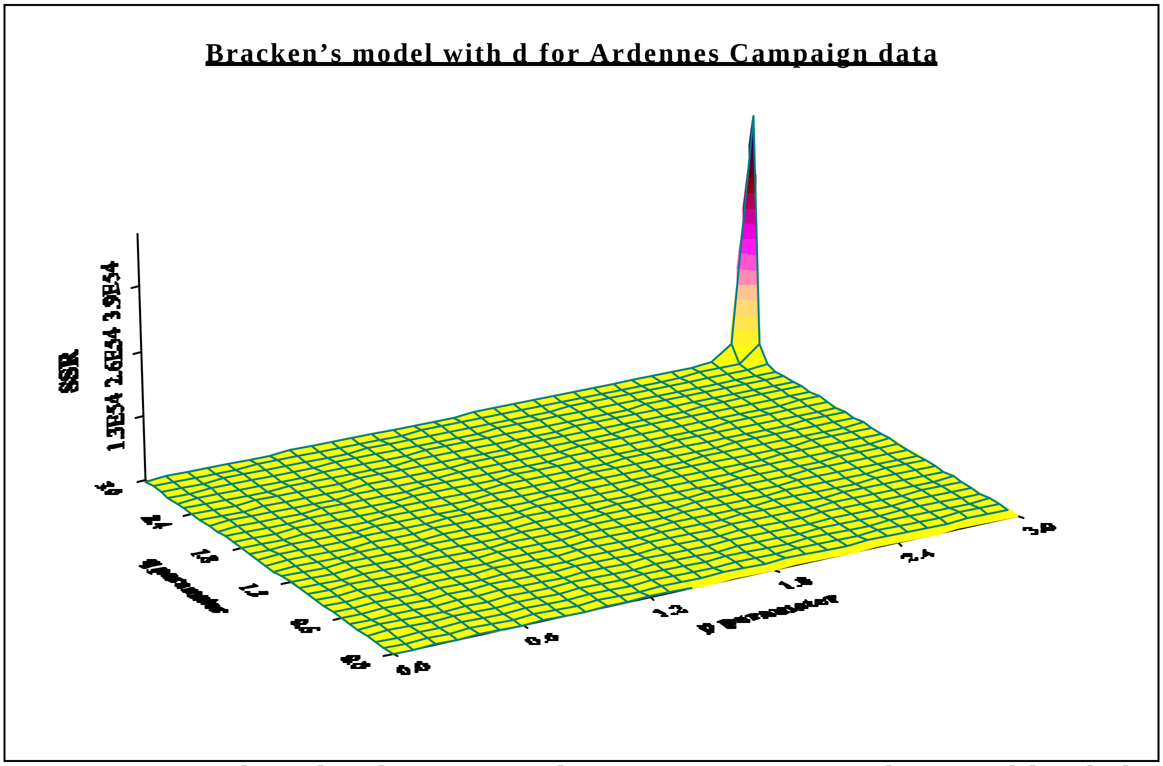

Figure 79. SSR values plotted versus p and q parameters using Bracken's model with the tactical parameter for Ardennes Campaign data.

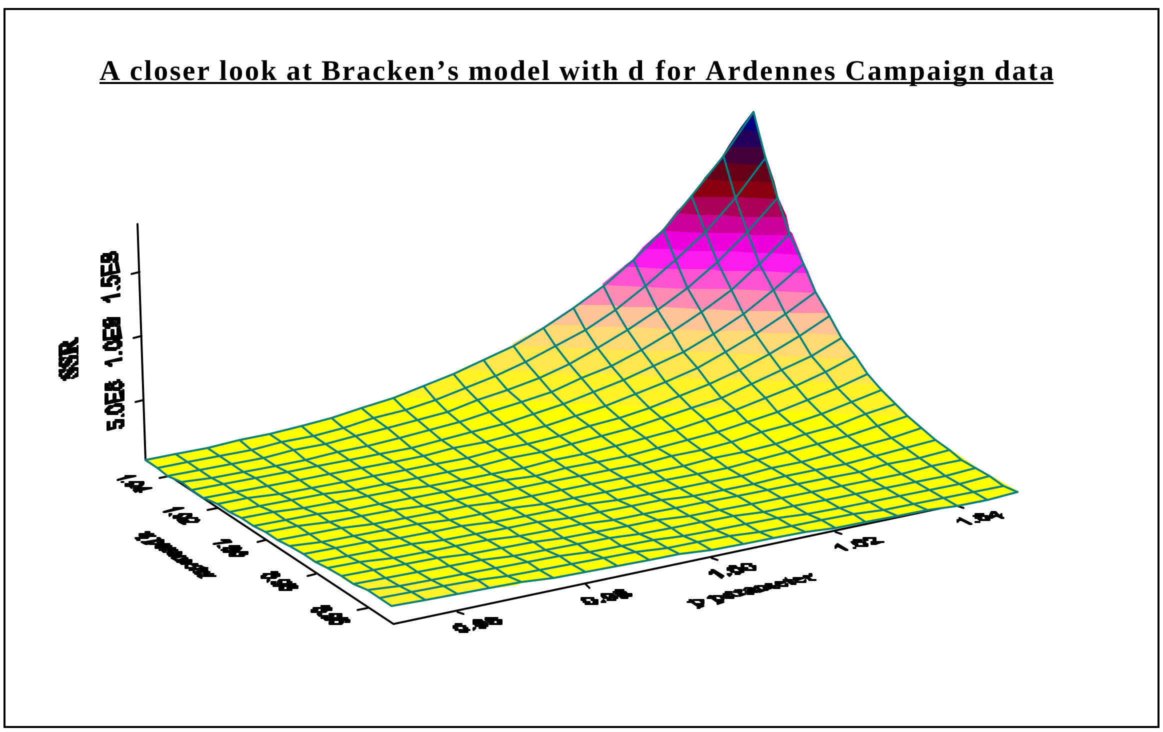

Figure 80. A closer look at the lowest SSR value for Bracken's model with the tactical parameter, for Ardennes Campaign data when p=1 and q=1.

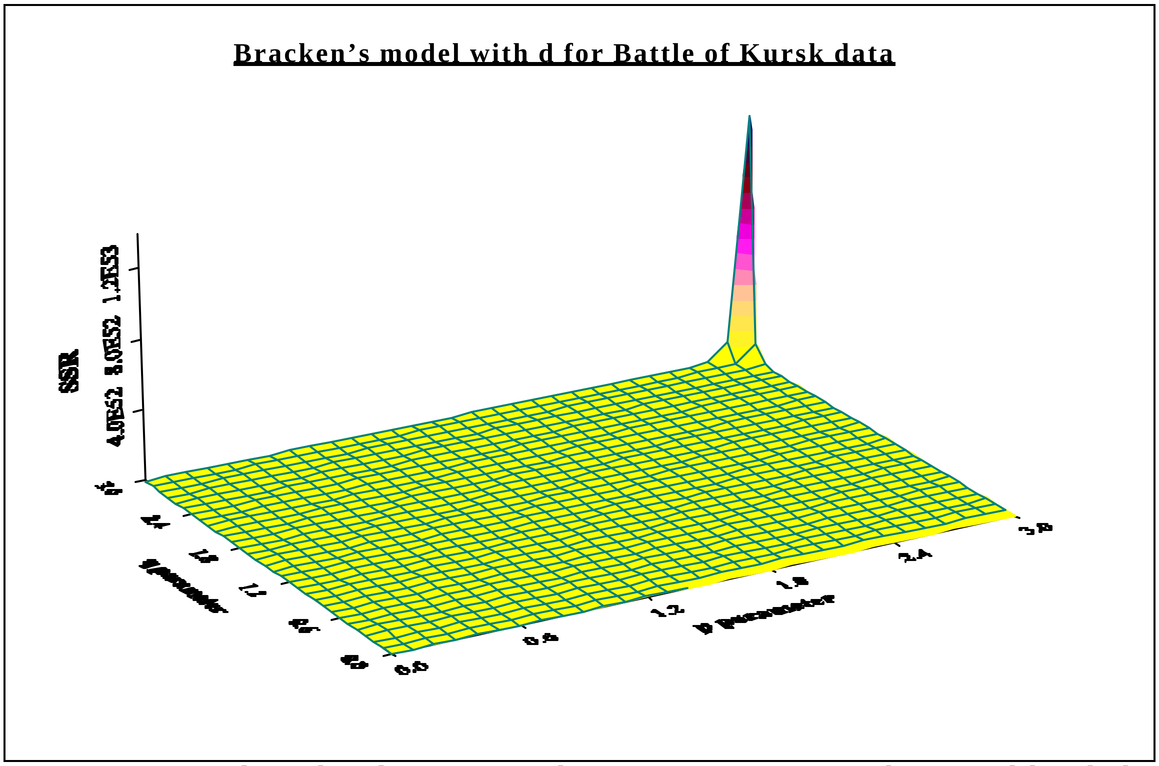

Figure 81. SSR values plotted versus *p* and *q* parameters using Bracken's model with the tactical parameter for Battle of Kursk data. This pattern holds true for other cases too.

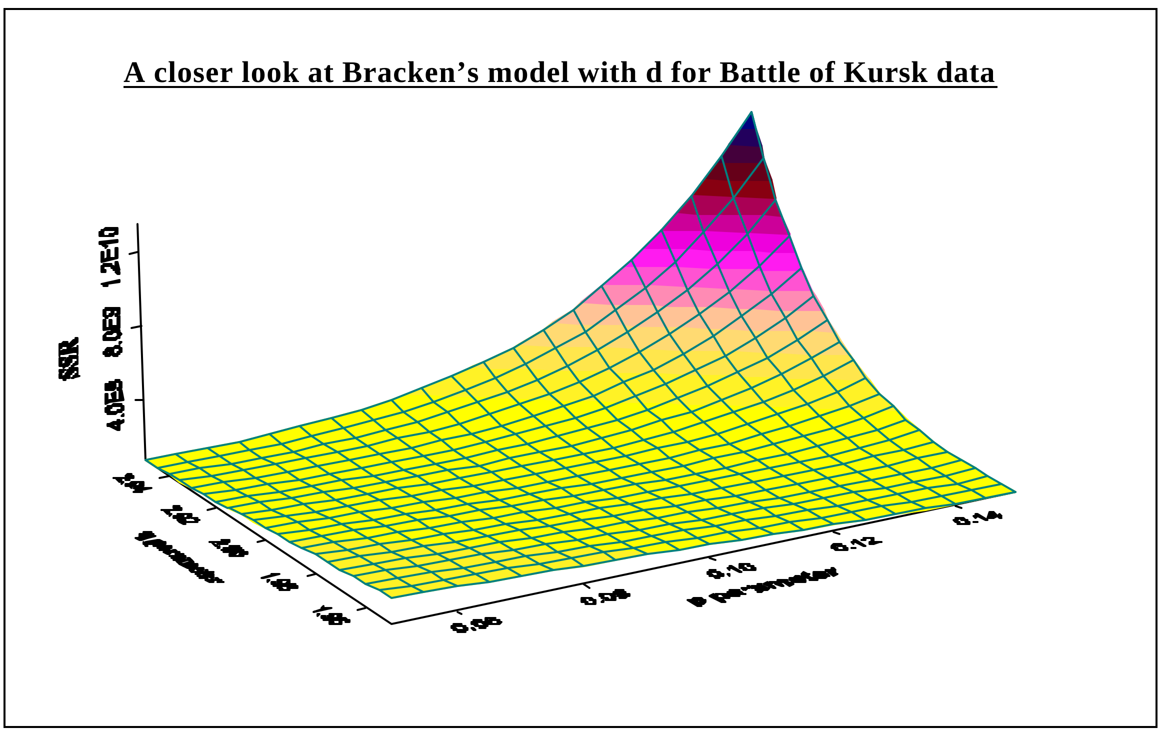

Figure 82. A closer look at the lowest SSR value for Bracken's model with the tactical parameter, for the Battle of Kursk data when p=0.1 and q=2.0. The observed pattern is similar to what is observed for Ardennes.

for regression through the origin given p and q. The best fitting a and b parameters are given as follows:

$$a = \frac{\sum_{i=1}^{n} \dot{B}(i)[R(i)^{p} B(i)^{q}]}{\sum_{i=1}^{n} [R(i)^{p} B(i)^{q}]^{2}} \tag{108}$$

$$b = \frac{\sum_{i=1}^{n} \dot{R}(i)[B(i)^{p} R(i)^{q}]}{\sum_{i=1}^{n} [B(i)^{p} R(i)^{q}]^{2}} \tag{109}$$

where i is the index of the days in a given battle, and n is the number of days in a given battle.

Figure 83 shows the 3-D plot of SSR values found for the Battle of Kursk data, where p values are varied between -0.5 and 10.0 with increments of 0.1, q values are varied between -1.0 and 3.0 with increments of 0.1, d=1.0, a and b values depend on p and q, and are determined by equations V.A.(108) and V.A.(109). Figure 85 shows the same area using a contour filled plot. A contour plot displays the contours of equally fitting p and q values in terms of SSR. This surface was generated with d fixed at 1.0. The models found in Fricker, Bracken and Clemens used a d parameter; hence their place on the surface does not necessarily measure the goodness of their fit. Furthermore, Fricker and Clemens used differently formatted data.

Figures 85 and 86 represent a detailed description of the region with the best fit. Figure 85 shows the 3-D plot of the SSR values found for the Battle of Kursk data, where p values are varied between 3.0 and 9.0 with increments of 0.1, q values are varied between 0.0 and 2.5 with increments of 0.1, d=1.0, a and b values depend on p and q, and

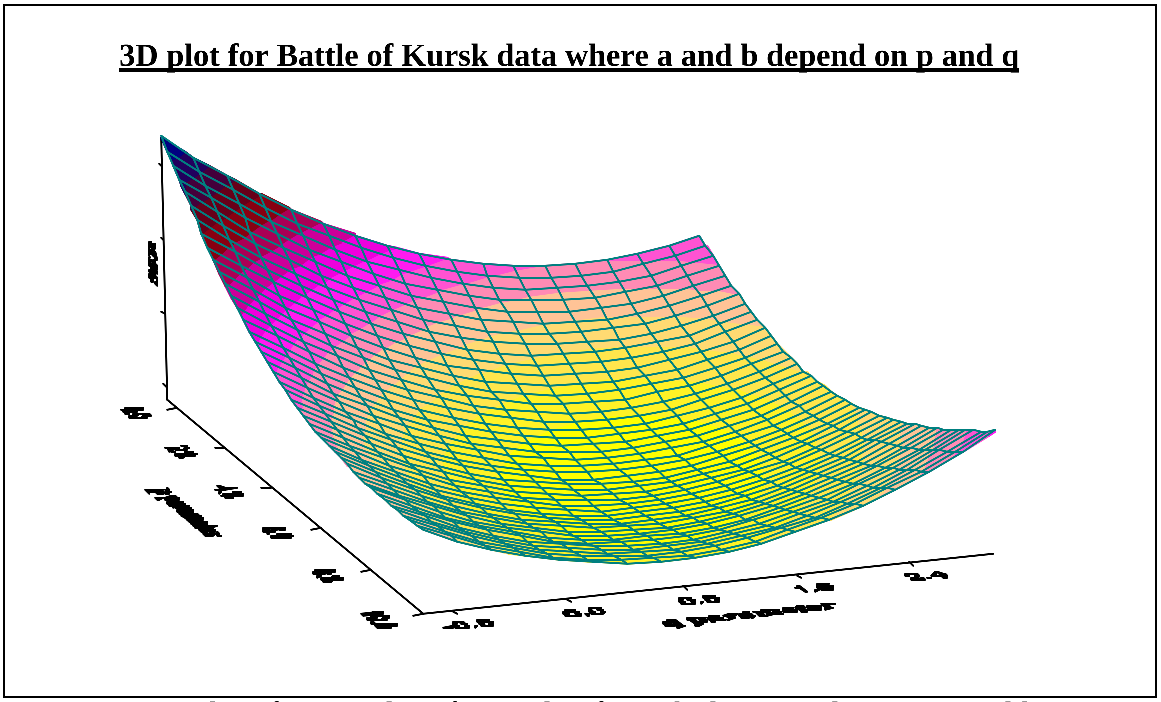

Figure 83. 3D plot of SSR values for Battle of Kursk data, *p* values are varied between –0.5 and 10.0 with increments of 0.1, *q* values are varied between –1.0 and 3.0 with increments of 0.1, d=1.0, *a* and *b* values depend on *p* and *q*.

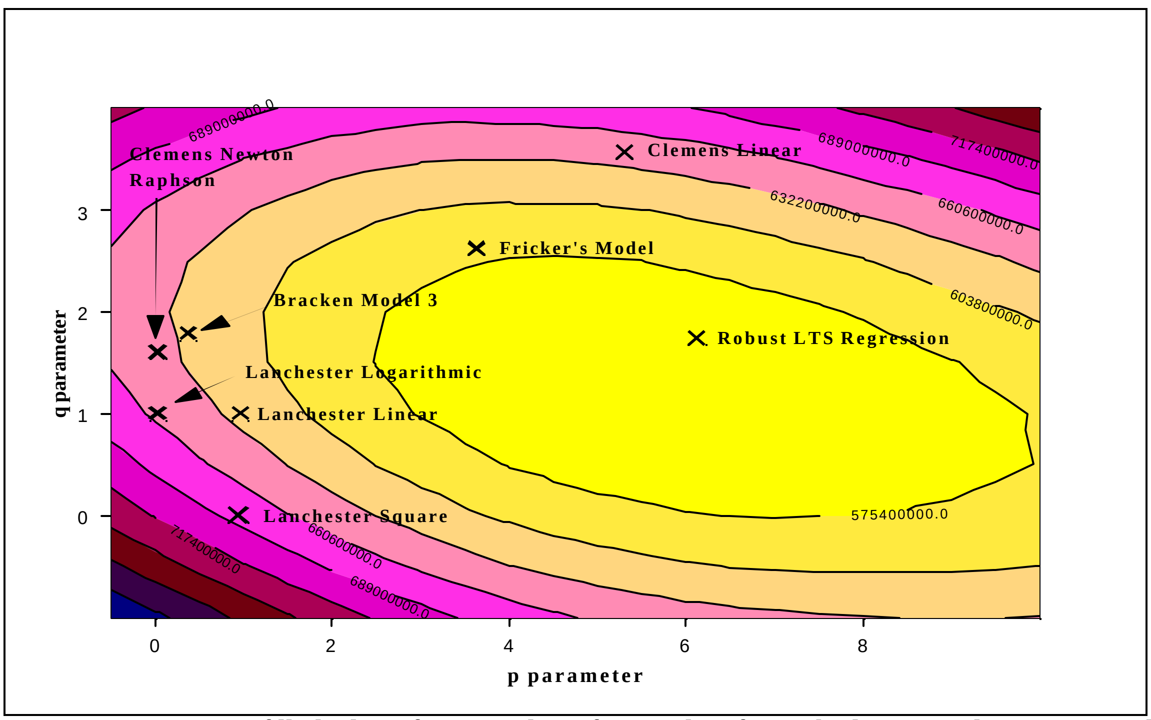

Figure 84. Contour filled plot of SSR values for Battle of Kursk data, *p* values are varied between –0.5 and 10.0 with increments of 0.1, q values are varied between –1.0 and 4.0 with increments of 0.1, d=1.0, *a* and *b* values depend on *p* and *q*. Also shown are each of the similar findings around the same area.

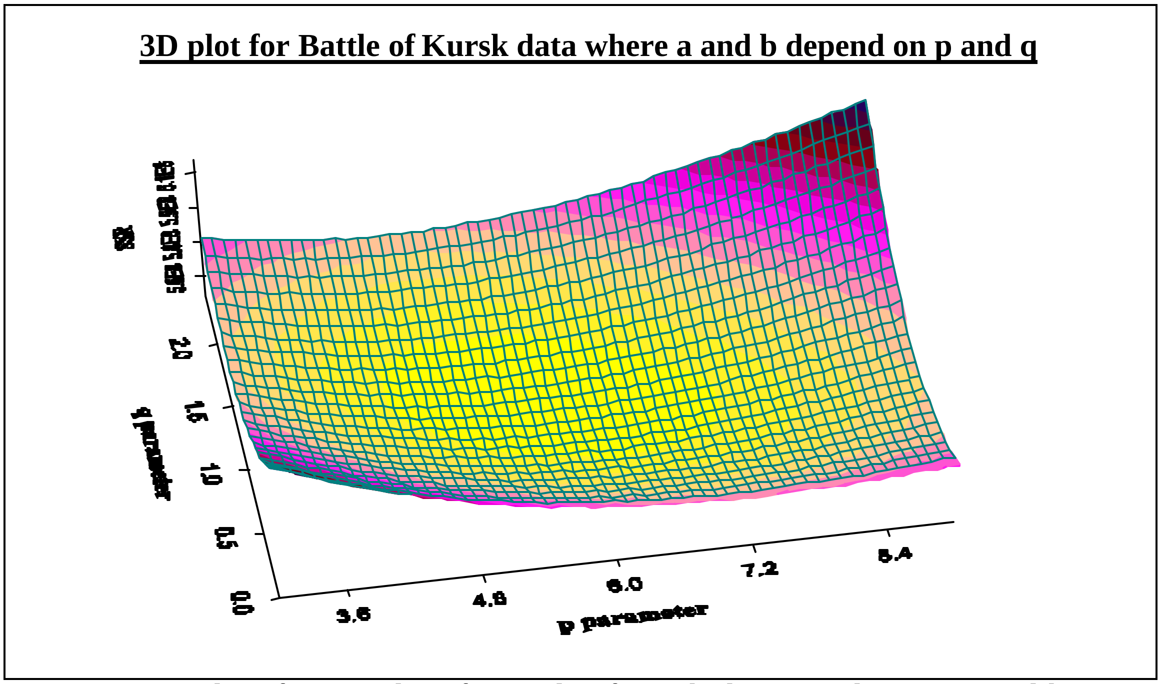

Figure 85. 3D plot of SSR values for Battle of Kursk data. *p* values are varied between 3.0 and 9.0 with increments of 0.1, *q* values are varied between 0.0 and 2.5 with increments of 0.1, d=1.0, *a* and *b* values depend on *p* and *q*.

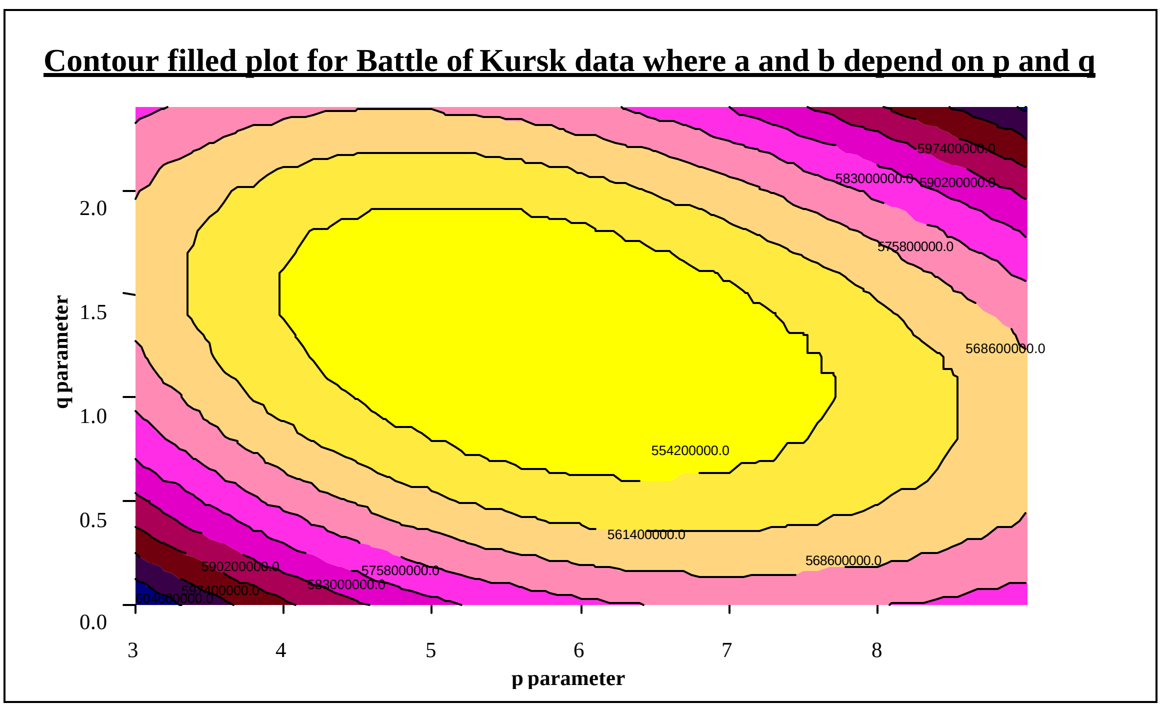

Figure 86. Contour filled plot of SSR values for Battle of Kursk data. *p* values are varied between 3.0 and 9.0 with increments of 0.1, *q* values are varied between 0.0 and 2.5 with increments of 0.1, d=1.0, *a* and *b* values depend on *p* and *q*.

are given as in equations V.A.(108) and V.A.(109). Figure 86 shows the same area using a contour filled plot.

The above results imply that there is no absolute best fit, as long as one stays in the broad vicinity of the identified best fit, it is likely to have similar fits, and there is not just one set of parameters that clearly gives a best fit. One can still find a similar fit as long as the estimated parameters are in the vicinity of the best fit. However, the area of the surface bounded by the lowest contour in Figure 86 says, roughly, the best fitting models have a q parameter between 0.5 and 2, while the p parameter is between 4 and 8. This observation is significant in that the Lanchester linear and square laws have a p value of 1 and the logarithmic law has a p value of 0. When p=1, the best fitting model has a 9% higher SSR value than the lowest found value of 5.54×10^{8}; which was found by using LTS regression.

A wide range of parameters fit equally well, but the question is, is this true for Ardennes campaign data too? Figure 87 shows the contour filled plot for the Ardennes data together with the best fits determined by Bracken and Fricker. Again, a and b depend on p and q, and d=1. The a and b parameters are chosen to minimize SSR for regression the through origin, and are given in equations V.A.(108) and V.A.(109). When Figure 87 is examined, one can see that the general pattern observed for the Kursk data is also observed for the Ardennes campaign data.

The bottom line conclusion is that different researchers using different methods all came up with very different answers because the surface around the models' fits is very flat.

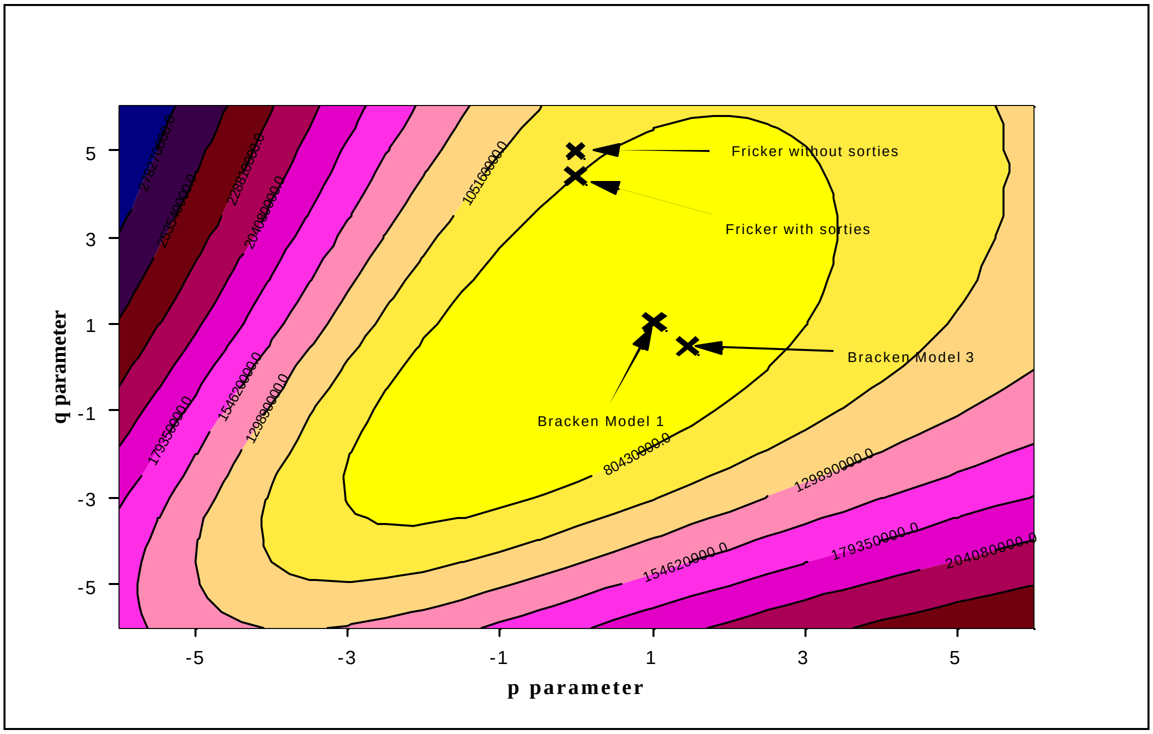

Figure 87. Contour filled plot of SSR values for Ardennes Campaign data. *p* values are varied between –6.0 and 6.0 with increments of 0.1, *q* values are varied between –6.0 and 6.0 with increments of 0.1, d=1.0, *a* and *b* values depend on *p* and *q*. Note: Fricker's method has a better R^2, this is not apparent from this figure which uses differently formatted data and no tactical parameter *d*.

- Fricker's methodology, when applied to Battle of Kursk data gives a better fit than Bracken's. This conclusion implies that the algorithm Fricker introduces in his study is useful in fitting the data and can be used in further studies.
- Throughout the study, with the exception of the two models whose results are given in equations IV.A.2.(14), IV.A.2.(15), IV.A.2.(18), IV.A.2.(19), and the models with the negative exponential parameters, the *a* parameter is always greater than the *b* parameter. This consistency implies that individually, German soldiers are more lethal than Soviet soldiers, and they also fought better than the Soviets. Also, according to this difference in parameters, German military expertise was much better than the Soviets' in the Battle of Kursk. This finding

was consistent throughout the battle. Despite their lack of military expertise, the Soviets won the battle due to their massive amount of supplies and manpower.

- Another significant result for the a and b parameters is that both are very small, and this result is consistent with Fricker's findings.
- The best fit to the data is observed when robust LTS regression model is applied. Robust LTS regression gives the smallest SSR value, which is 5.54×10^{8} when d=1.0. This finding was significant because it indicates no attacker/defender advantage.
- The d parameter, which gives the best fit using the linear regression model, is found to be 1.17. Using the a, b, p and q parameters found in equations IV.B.3.b.(40) and IV.B.3.b.(41), the data is analyzed in four distinct periods. The analysis revealed that it was usually advantageous to be the attacker in Battle of Kursk campaign. The only two days when the defender had the advantage was the first day when Germans attacked and the eighth day when the Soviets attacked. The Battle of Kursk was a major tank battle. Since a tank is an assault weapon, and is not optimally used as a defense weapon, the rationalization that the attacker will always have the advantage is considered a natural outcome of this battle.
- Finding only one tactical parameter d, and refusing to vary from that parameter through the battle is apparently a mistaken approach. The tactical parameter for a battle in which one side attacks a defender behind heavily fortified positions must not be the same with the tactical parameter for a battle in which one side is counterattacking and the other is making a hasty defense.

- The tactical parameter in the first half of the battle, being greater than the tactical parameter in the second half, indicates that the Germans were better than Soviets both in attack and defense. This finding is especially consistent with the tactical parameter value of 0.32 found on the eighth day, during which the Soviets counterattacked.
- The plots investigated in Section IV.B.7 show that if the force ratio is higher, then loss will be reduced because, as force ratio increases, loss decreases. This result is consistent with the force ratio approach, which is widely used in military simulation models today, showing the effectiveness and validity of the approach.
- The R^2 values given in Table 32 indicate that the model with the change point 7/7 represents the data with the best fit, with an R^2 value of 0.7748. The second best fit is observed with the model that divides the campaign in four different parts, with an R^2 value of 0.5689. This suggests that even an individual battle cannot be viewed as homogenous.
- Some models have negative R^2 values, meaning that one can have a better estimate of the attrition just by using the mean value, as opposed to using the model itself, and going into the modeling business. In other words, it is better to use the mean value for estimating the attrition instead of using the estimate given by the models, which have negative R^2 values. The negative R^2 values found in section IV.A.2 for Fricker's models occurred because the parameters were rounded off in Fricker [Ref.6]. Had more precise values been available the R^2 values would have been positive.

- Throughout the thesis, the robust LTS regression technique gives better fits to the data than the linear regression technique. This is because the robust regression models are useful for fitting linear relationships by discounting outlying data when the given data in hand contains significant outliers, as in our case.
- Combat models cannot provide clear-cut results to a military analyst. One cannot determine the outcome of a battle precisely by using combat models. Together with their use to gain insight about the battles and campaigns that happened in the past, combat models help to make better decisions by enabling the decision-maker to compare different alternatives using various combat modeling techniques.

B. RECOMMENDATIONS

The models presented in this thesis study do not include nor analyze total manpower data. Data for total manpower is present in the KDB and can be examined in the future studies.

The weights used for aggregating the forces are subject to research. A more complex model, one that includes the weights of weapons systems as unknown parameters to be estimated, can be set up and analyzed to find a better fit. And when the complex and numerous different weapon systems of today's military are considered, this shows potential to be a very interesting research topic.

Weapon systems other than the featured tanks in this study can be used to find a model with a better fit using a homogenous weapons scenario.

In this thesis, the change points for each side is on the same day. Another way to find a better fit would be to use different change points for each side, rather than using the same change point.

A final recommendation for the continued analysis of the Kursk database is to try to fit additional models other than the Lanchester models.