I. INTRODUCTION

War is a conflict between nations or states carried on by force of considerable duration and magnitude, by land, sea, or air for obtaining and establishing the superiority and dominion of one over the other for some cause. It is defined more concisely as the state of usually open and declared armed hostile conflict between states or nations [Ref.1]. When these conflicts reach global proportions, they are known as world wars. Among the causes of war are ideological, political, racial, economic, and religious conflicts. According to Karl von Clausewitz, war is a "continuation of political intercourse by other means" and often occurs after means of compromise and mediation have failed.

Throughout history, war has been a topic of analysis for scientists and researchers, especially following World War II. In the shadow of a possible outbreak of nuclear war between the United States and Russia, more research has been done on the subject of war than ever before.

A. COMBAT MODELING

This study, instead of analyzing the concept of war at large, will analyze and focus on a smaller part of war, which we usually name *combat*, *battle* or *campaign*. Even though these terms are generally used as if they had the same meaning, the word *combat* is used for defining active, armed fighting between two enemy forces, while the word *battle* is used for defining a hostile encounter or engagement between opposing military forces. The word *campaign* is used for defining military operations for a specific objective, and defines a connected series of military operations aimed at accomplishing a specific operational and strategic objective. A *campaign* forms a separate and distinct phase of war, and it is this “small” part of war that is the concentration of this study.

Throughout history, combat has been an important topic of analysis, just like war itself. Scientists, researchers, and the military have tried to understand and estimate beforehand the nature of combat in order to formulate some theory about its dynamics and most importantly, its outcome. Researchers who studied combat modeling and attrition were aware of the influence their studies could have on the outcome of a battle. A natural consequence of these studies was the emergence of combat models in the beginning of the early 20th century.

Attrition is a reduction or decrease in number, size, or strength of a force and is at the core of every general discussion of warfare. The term attrition defines a wearing down or weakening of resistance, especially as a result of constant harassment, abuse, or attack.

Soviets argue that Osipov [Ref.2] was the first to study and discover the equations most often used when modeling attrition in combat. The equations are widely known as, “Lanchester’s equations.” Regardless of claims of prior or parallel discovery, Lanchester’s equations for attrition provided the origin for modeling attrition in the United States and around the world.

Frederick William Lanchester (b.1868, London, England; d.1946, Birmingham, Warwickshire) was an English automobile and aeronautics pioneer who built the first British automobile in 1896. Lanchester's interest in aeronautics was first expressed in a paper he wrote in 1897, a work ahead of its time discussing the principles of heavier-than-air flight. Between 1907-1908, he published a two-volume work embodying distinctly advanced aerodynamic ideas. As a member of the Advisory Committee on Aeronautics in 1909 and, later, as a consultant to the Daimler Motor Company, Ltd., Lanchester also contributed to the development of the field of operations research. [Ref.3].

Lanchester proposed that attrition could be mathematically modeled, and introduced his equations as a means of investigating the future impact that the recently invented airplane might have on the nature of warfare [Ref.4]. Thus, at the beginning of World War II, Lanchester equations and other differential equations of a similar nature were known to some of the scientists who later became active in operations research [Ref.5].

Today, with the advent of computers, Lanchester-based models of warfare are widely used in the decision making process for research, development, acquisition of weapons systems, force mix decisions, and for aiding in the development of operational plans.

B. LANCHESTER EQUATIONS

As described in Fricker [Ref.6], the basic generalized Lanchester Equations are of the form:

$$\dot{B}(t) = aR(t)^p B(t)^q \tag{1}$$

$$\dot{R}(t) = bB(t)^p R(t)^q \tag{2}$$

where *B(t)* and *R(t)* are the strengths of blue and red forces at time t, $\dot{B}(t)$ and $\dot{R}(t)$ are the rates at which blue and red force levels are changing at time t, a and b are attrition parameters, p is the exponent parameter of the attacking force, and q is the exponent parameter of the defending force. The model begins with initial force sizes, *B(0)* and

R(0), that, when solved numerically, are incrementally decreased according to the relationship $B(t+Dt) = B(t) - Dt\,\dot{B}(t)$ and $R(t+Dt) = R(t) - Dt\,\dot{R}(t)$. In an equally matched battle, where the ratio of the forces stays constant over time, $B(t)/R(t) = \dot{B}(t)/\dot{R}(t)$, for all t. This is equivalent to the condition that $bB(t)^{p-q+1} = aR(t)^{p-q+1}$ for some p and q, and all t.

Two versions of the Lanchester equations are of particular interest. When $p = q = 1$ (or, more generally, when p-$q = 0$) force ratios remain equal if $aR(0) = bB(0)$, and hence this condition is called, *Lanchester's linear law*. The interpretation of Lanchester's linear law is that a battle governed by this model is characterized as a collection of small engagements, and was proposed by Lanchester [Ref.4] as a model for ancient warfare. The equation is also considered a good model for area fire weapons, such as artillery [Ref.7].

Lanchester contrasted the Linear Law with the condition $p = 1$, $q = 0$ (or, more generally, p-$q = 1$), which is called *Lanchester's square law*, where the force ratios remain equal when $aR(0)^2 = bB(0)^2$. He theorized that the square law applies to modern warfare, in which both sides are able to aim their fire. His model suggests that in modern warfare, combatants should concentrate their forces.

A third version with $p = 0$, $q = 1$ (or, more generally, q-$p = 1$) is called *Lanchester's logarithmic law*.

C. THESIS OUTLINE

This thesis consists of five chapters. This first chapter introduces the general concept of combat modeling and the widely used Lanchester Equations. The second chapter reviews previous studies on combat modeling. The Battle of Kursk data is also introduced in this chapter, and the study methodology for the thesis is explained. The third chapter briefly covers the history of Battle of Kursk, and explores and analyzes the battle's data in depth to gain insights before attempting to fit models to it. Additional details about the personnel and weapon systems data is given in Appendix A.

The primary objective of Chapter Four is to find the best model that fits the Battle of Kursk data. To accomplish this objective, the methods of previous studies are applied, and then all new exploratory models are implemented. The results derived from the regression analysis methods are briefly evaluated in this chapter. The fifth chapter interprets the results that are derived from the regression analysis methods in Chapter Four. Chapter Five contains the final conclusions and recommendations implicated by results, and also mentions future areas of study on combat modeling.