EXECUTIVE SUMMARY

War is a conflict between nations or states carried on by force of considerable duration and magnitude, by land, sea, or air for obtaining and establishing the superiority and dominion of one over the other for some cause. Throughout history, war has been a topic of analysis for scientists and researchers, especially following World War II.

Soviets argue that Osipov [Ref.2] was the first to study and discover the equations most often used when modeling attrition in combat. The equations are widely known as "Lanchester's equations." Regardless of claims of prior or parallel discovery, Lanchester's equations for attrition provided the origin for modeling attrition in the United States and around the world. Today, with the advent of computers, Lanchester-based models of warfare are widely used in the decisionmaking process for research, development, acquisition of weapons systems, force mix decisions, and for aiding in the development of operational plans.

The basic generalized Lanchester Equations are of the form [Ref.6]:

$$\dot{B}(t) = aR(t)^p B(t)^q \tag{1}$$

$$\dot{R}(t) = bB(t)^p R(t)^q \tag{2}$$

where *B(t)* and *R(t)* are the strengths of blue and red forces at time t, $\dot{B}(t)$ and $\dot{R}(t)$ are the rates at which blue and red force levels are changing at time t, a and b are attrition parameters, p is the exponent parameter of the attacking force, and q is the exponent parameter of the defending force.

Two versions of the Lanchester equations are of particular interest. When $p = q = 1$, force ratios remain equal if $aR(0) = bB(0)$, and hence this condition is called, *Lanchester's linear law*. The interpretation of Lanchester's linear law is that a battle

governed by this model is characterized as a collection of small engagements, and was proposed by Lanchester as a model for ancient warfare. The equation is also considered a good model for area fire weapons, such as artillery. Lanchester contrasted the Linear Law with the condition $p = 1$, $q = 0$, which is called *Lanchester's square law*, where the force ratios remain equal when $aR(0)^2 = bB(0)^2$. He theorized that the square law applies to modern warfare, in which both sides are able to aim their fire. His model suggests that in modern warfare combatants should concentrate their forces. A third version with $p = 0$, $q = 1$ is called *Lanchester's logarithmic law*.

Past empirical validation studies of Lanchester Equations include the work of Bracken on the Ardennes campaign of World War II, Fricker, also on the Ardennes campaign, Clemens on the Battle of Kursk of World War II, and Hartley and Helmbold on the Inchon-Seoul campaign of the Korean War. These works are among the few quantitative studies that use daily force size data for real battles.

Bracken formulated four different models for the Ardennes campaign, which are variations of the basic Lanchester equations, and estimated their parameters for the first ten days of the of the Ardennes campaign (December 15, 1944 through January 16, 1945). He concluded that: (1) the Lanchester linear model best fits the Ardennes campaign data, (2) when combat forces are considered, allied individual effectiveness is greater than German individual effectiveness, (3) when total forces are considered, individual effectiveness is the same for both sides, and (4) there is an attacker advantage throughout the campaign.

Fricker's paper revisited Bracken's modeling of the Ardennes campaign, using linear regression to fit the total body of data from the entire campaign, including air sortie

data. Fricker concludes by saying that one side's losses are more a function of his own forces than a function of the opponent's forces, like the logarithmic law, and gives the Gulf War as an example to support this theory.

Clemens' analysis examined the validity of the Lanchester Models as they are applied to modern warfare using data from the Battle of Kursk. His analysis is an extension of Bracken's and Fricker's analyses of the Ardennes Campaign. He concludes that the Lanchester logarithmic model in both scalar and matrix form fits better than the Lanchester linear and square models.

Hartley and Helmbold tested Lanchester's square law using the data from the Inchon-Seoul campaign. They conclude that: (1) the data do not fit a constant coefficient Lanchester square law, (2) the data better fit a set of three separate battles (one distinct battle every six or seven days), (3) Lanchester's square law is not a proven attrition algorithm for warfare (but neither can it be completely discounted), and (4) more real combat data are needed to validate any proposed attrition law.

This thesis takes a closer look at Lanchester's equations using recently available data on the battle of Kursk. In July 1943, the Battle of Kursk, the largest tank battle in history, took place around the city of Kursk, Russia, and ended in the defeat of the Germans. A detailed database of this battle was recently developed. The data were collected from military archives in Germany and Russia by the Dupuy Institiute (TDI), and are reformatted into a computerized data base, designated as the Kursk Data Base (KDB). KDB is recently documented in the KOSAVE (Kursk Operation Simulation and Validation Exercise) study. The data are two-sided, time phased (daily), and highly detailed. They cover 15 days of the Battle of Kursk.

A total of 39 diverse models are fit to the Battle of Kursk data using different approaches. These approaches include applying the methodologies of previous studies, using robust LTS (least trimmed squares) regression for estimation purposes, including air sortie data of the battle, considering the battle in separate phases, using different weights to aggregate the data, fitting basic Lanchester equations, fitting Morse-Kimball equations, and applying parameters found in previous studies.

The findings from this research include:

- It is observed that the original Lanchester equations do not fit to the Battle of Kursk data, and therefore may not be appropriate for modeling the combat. Of the three ill-fitting Lanchester equations, the best fit is obtained by applying the linear law, which is used for modeling ancient warfare or area fire.
- The parameters derived from Bracken and Fricker's Ardennes studies do not apply to the Battle of Kursk data. This implies that there are no unique parameters that apply to all battles.
- Throughout the study, the *a* parameter is generally greater than the *b* parameter. This implies that individually German soldiers were more lethal than Soviet soldiers.
- The best fit to the data is observed when a robust LTS regression model is applied. The best fit occurred with no attacker/defender advantage.
- In the battle of Kursk, except for the first and eighth days, it was advantageous to be the attacker.
- The different approaches give very different estimates on the best fitting Lanchester parameters (p, q, a, b); see Table 1 for a sample of the range of

parameters found. A closer investigation reveals that the surface of the sum squared residuals (SSR), from the differences between the daily estimated and real losses over the battle, is very flat. This explains why such diverse answers are in the literature. Figure 1 shows a contour filled plot of SSR values for Battle of Kursk data with *p* values varied between –0.5 and 10.0 and q values varied between –1.0 and 4.0, and *a* and *b* values determined to minimize SSR given *p* and *q*. Different researchers, using different methods, all came up with completely different answers because the surface around the models' fits is very flat. Therefore, small changes in handling the data and the application of different estimation methodologies results in dramatically different parameter estimates. Thus, there is not enough data from the battle of Kursk, and Ardennes too, to differentiate between a wide range of Lanchester models. Unfortunately, none of the basic Lanchester models, liner, square, and logarithmic, provide a good fit.

Name of the model	***a***	***b***	***p***	***q***
Bracken Ardennes Model 1	8.0E-9	1.0E-8	1.0	1.0
Bracken Ardennes Model 3	8.0E-9	1.0E-8	1.3	0.7
Fricker Ardennes Com.Manpwr.w/o sortie	4.7E-27	3.1E-26	0.0	5.0
Clemens Kursk Linear Regression	6.92E-49	6.94E-48	5.3157	3.6339
Clemens Kursk Newt.-Raphson iteration	3.73E-6	5.91E-6	0.0	1.6178
Kursk Robust LTS Regression	2.27E-40	1.84E-41	6.0843	1.7312

Table 1. Results for the 6 of the 39 models explored in the study. Notice the wide range of parameter estimates.

- Combat models cannot provide clear-cut results to a military analyst. One cannot determine the outcome of a battle precisely by using combat models. Together with their use to gain insight about the battles and campaigns that happened in the past, combat models help to make better decisions by enabling the decision-maker to compare different alternatives using various combat modeling techniques.

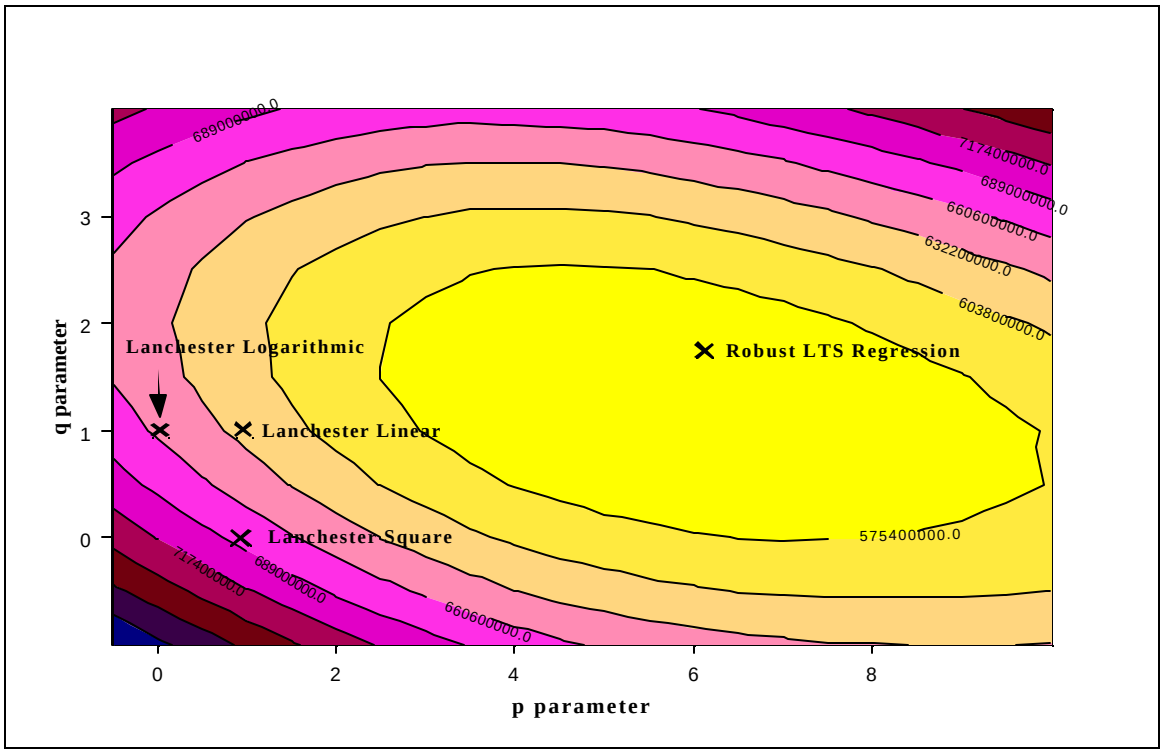

Figure 1. Contour filled plot of SSR values for Battle of Kursk data with no attacker/defender advantage considered. A wide range of diverse generalized Lanchester models give about the same fit.