MOES & Utility Theory Agenda

- Combat models as big functions
- Outputs of combat models
  - –MOES, MOPS, and MOOs (plus MoM and FoM)
- Comparing Distributions of Outcomes
- Utility Theory

A Model/Simulation as a Big Function

Model

$$\vec{Y} = F(\vec{X}) + \vec{\varepsilon}$$

Error

Output

Input & Model Parameters

- **With this view we can use standard statistical (e.g., CIs and HTs) & optimization (Steepest assent) techniques in analysis**

- **We often report out and make decision based on comparisons of functions of the output for given input sets (e.g., blue casualties as a function of various tactics in a given scenario).**

A Brief Aside on the Inputs X's

- Combinatorics make comprehensive analysis difficult
  - $10^{\wedge 30}$ is forever
  - Thousands of variable inputs (many of them uncertain)
  - Example: compare alternatives A, B, and C
    - » location (SWA, NEA, Europe)
    - » conditions (day/night, fog/clear)
    - » tactics (attack/defend, hasty/prepared)
    - » enemy resolve (stiff, nominal, minimal)
    - » other uncertainties (weapons, information, reliability, etc.)
    - » replications if Monte Carlo
- Result: Can't do all cases
  - Most likely
  - Bounding/stressing
  - Fancy statistical designs (see design of experiments)

MOPs and MOEs, Plus MOOs and More

- **Measure of Effectiveness (MOE):** A *quantitative* measure, generated by the model, that is used to compare the effectiveness of alternatives (e.g, systems, tactics, organizations) in achieving *operational objectives* (e.g, # of blue casualties, advance rate, fractional exchange ratio (FER), tons arriving, etc.).
  - Think: how much better do ***we*** (the force) do

- **Measure of Performance (MOP):** A quantitative (typically) system level measure of performance (e.g., max detection range, missile speed, task completion time, etc.). Typically lower-level (more detailed) measure than MOE (more later).
  - Think: how much better do ***I*** (the system) do
- **Measure of Outcome (MOO):** A metric that defines how operational requirements contribute to end results at higher levels, such as campaign or national strategic outcomes (i.e., did we win the battle/war).
- **Other terms…Measure of Merit (MoM), Figure of Merit (FoM)**

Properties of MOEs

- **Measure of Effectiveness (MOE) should be:**
  - Measurable
  - Quantifiable
    - scales of measurement
      - nominal (put into classes (e.g., service, color))
      - ordinal (rank order (A > B))
      - interval (A is 3.14 more than B)
      - ratio (A is 1.2 times bigger than B)
        - (be careful with ratios)
  - **Relate to operational objectives***

An Example

202 THE OBJECTIVE OF THE OPERATION

Finding the *real* problem is often one of the most difficult features of operational problems, as exemplified in the following case. Early in World War II a great number of British merchant vessels were being sunk or seriously damaged by Axis aircraft attack in the Mediterranean. The obvious answer was to equip these ships with antiaircraft guns and gun crews. This was done at great expense of men and equipment badly needed elsewhere. Questions concerning the soundness of this allocation of scarce resources were raised when reports showed that the gun crews were shooting down only about 4 percent of all attacking aircraft. This was a poor showing and seemed to indicate that the AA guns and crews were not worth the cost of installation. On more careful consideration, it was realized that the guns were not there primarily to shoot down German or Italian aircraft, but their objective was to protect the merchant vessels. And, in fact, as figures were accumulated, it became apparent that the AA guns and crews were doing the job rather well; of the ships attacked, 25 percent of those without protection had been sunk, while only 10 percent of the ships with protection were lost in the same period. [1]

I

WE

The objective being considered here is that of the person who has the ultimate responsibility for making the decision. A commanding officer's objective may often be determined by studying the mission given him by his superior. A business executive's objective is largely determined by, or at least consistent with, the purpose and goals of his organization.

From a Chip to the DoD

- Let’s right measures at each level in the hierarchy (What I do)
  - » Computer board
  - » Fire control radar
  - » Gun/missile system
  - » Ship
  - » Task Force
  - » Fleet
  - » Navy
  - » DoD

Parry’s Rule

- Write down the MOEs in the hierarchy (think “I” at each stage), really need to do at and above the question being asked.
  - » Computer board
  - » Fire control radar
  - » Gun/missile system
  - » Ship
  - » Task Force
  - » Fleet
  - » Navy
  - » DoD

Parry’s Rule Continued

- How to find the appropriate MOEs
  - » Form hierarchy at and above the level that the question is being asked
  - » Write down the measures (“I”) at each stage in the hierarchy. “I” part is system or thing we are interested in.
  - » Determine the “we” part = force in which I’m going to imbed the “I” system
  - » The MOEs for “I” are usually the MOPs for “I” at one or (more typically) two levels above where the question is being asked.
    - – For FC Radar MOE involves missile/ship measures
    - – E.g., For missile MOE involves ship/task force measures
  - » Need to model/simulate/analyze at the “we” level

Some Guidance on MOEs

- Must reflect operational objectives
- Must be quantitative (best if objective)
  - Be wary of ratios
  - Avoid dimensionless numbers
- Decide on them early in the study (definitely not after the fact)
- Keep them simple, understandable to decisionmaker
- Make bigger better (or, at least be consistent)
- Be careful mixing cost and effectiveness, often fix one and iterate.
  - Army (TRADOC=effectiveness, AMC=cost)
- Context is everything
  - Synergism (e.g., blue tank effectiveness might depend on whether there is a minefield)
  - Can't separate from scenarios and missions
    - » 19 losses in a company versus division, peace keeping versus war

Some Popular MOEs

- Traditional
  - Fractional Exchange Ratio (FER)
    - » FER = (Red killed/total Red)/(Blue killed/total Blue)
    - » If > 1 Blue will win a fight-to-the-finish
  - Forward Edge of Battle Area (FEBA) movement
    - » Less relevant with non-linear battlefields

- More typical in recent studies
  - Time to complete the mission
  - Number of Blue casualties
    - Collateral risks

Another Example
(notional, from the Rockower notes)

- Sonobuoy selection
  --Suppose there are several candidates
- MOE1 = radius of coverage (R)
- MOE2 = sweep width (2R)
- MOE3 = coverage area = πR^2
- MOE4 = Pr(sink a type of target)

- Definition: If MOEi = H(MOEj), where H is a monotonic function, then the MOEs are *Decision Equivalent*.

What if we Get a Probability Distribution on Effectiveness

- Sonobuoy selection continued (performance depends on "random" acoustic conditions
- Sonobuoy #1
  - R = 0 nm with Pr(.5)
  - R = 4 nm with Pr(.5)
- Sonobuoy #2
  - R = 2.5 nm with Pr(1)
- Which is preferred?
  (will depend on context (e.g., what if need 3 mile detection to intercept sub trying to penetrate a barrier)

- Note: ranking of alternatives is more difficult with probabilistic outcomes
  (I.e., $E[R_1] < E[R_2]$, but $E[\pi R_1^2] > E[\pi R_2^2]$)

- May need *utility theory* to consistently order probabilistic outcomes
  - Sometimes can order through ***stochastic dominance***.
  - X stochastically dominates Y if for all a Pr(X>a) > Pr(Y>a)

Ordering Probabilistic Outcomes

- Definition: A ***lottery*** (L) is an uncertain proposition with specified probabilities and outcomes.
- Example 1: lottery ticket
  - action a1, don't buy $1 ticket (value is $1 with Pr=1)
  - action a2, buy $1 ticket (outcome is 0 with $Pr=1.-10^{-7}$, and $5x10^{6}$ with $Pr= 10^{-7}$
  - What are the expected returns?
  - Does it make sense to buy a ticket?

- Note: depends on the person (I.e., utility is personalistic)
- If you buy lottery tickets (i.e., prefer action 2 to action 1) you are (in this situation) ***risk preferring!***
  - That is, you favor an action with lower expected return
- Note: E[L] = $\sum_{\text{possible outcomes(i)}} pr(outcome=i) \times (\text{payoff of i})$

Another Example

- Example 2: two bets (flip coin)
  - action a1:
    >> if heads (get \$60 with Pr=.5)
    >> if tails (get -\$40 with Pr=.5)

    action a2:
    >> if heads (get \$55,000 with Pr=.5)
    >> if tails (get -\$45,000 with Pr=.5)

- If you prefer action 1 you are (in this decision) *risk averse*!
  - Again, you favor an action with lower expected return

- Note: this is a good example that “scale matters”
  - If scale matters then YOU have a non-linear utility function

- Of course, peoples value of money depends on how much they have.

The Selling Price of a Lottery

- The ***<u>Selling Price</u>*** (SP) of a Lottery (L) is the minimum amount you would have to be paid to give up the lottery.
- Let's do some examples

- If SP(L) > E[L], you are risk preferring
- If SP(L) < E[L], you are risk averse
- If SP(L) = E[L], you are risk indifferent

- Note: If SP(L) = E[L], then you are an "expected value" decision-maker I.e. SP(L) = E(L), and SP(L1) > SP(L2) <==> E[L1] > E[L2]
- Can depend on scale (draw picture)
- Note: can do similar stuff with ***<u>buying price</u>***.

Going to the limit: “The Saint Petersburg Paradox”

- I will flip a fair coin until I get a tail.
- If I get a tail on the first flip you receive 2 cents. If I get my first tail on my second flip you get $2^{\wedge 2}$ cents. If I get my first tail on the nth flip you get $2^{\wedge n}$ cents.
- How much would you be willing to pay to play such a game?
- Are you an “expected value” decision-maker with respect to money?

Utility Functions

- First, some notation

  - If a lottery L has outcomes $(o_1, o_2, \ldots, o_n)$ with corresponding probabilities $(p_1, p_2, \ldots, p_n)$, with $\Sigma p_i=1$ and all p_i 0, then we write L as $[p_1,o_1;\ p_2,o_2\ ;\ \ldots\ ;\ p_n,o_n]$

  - If we have two outcomes o_i and o_j ,
    >> we write o_i is preferred to o_j by $o_i > o_j$
    >> we write o_i is indifferent to o_j by $o_i \sim o_j$
    >> we write o_j is <u>not</u> preferred to o_i by o_i o_j

- Four axioms of Utility
  - Ordering of alternatives and transitivity
  - Continuity
  - Monotonicity
  - Decomposable

The Axioms for Utility Functions

- Ordering of alternatives and transitivity
  -- For any two outcomes o_i and o_j, one of the following must be true: $o_i > o_j$ or $o_i < o_j$ or $o_i \sim o_j$ (that is I must be able to order any two preferences). And, (1) If $o_i \quad o_j$ and $o_j \quad o_k$, then, $o_i \quad o_k$, and (2) If $o_i \sim o_j$ and $o_j \sim o_k$, then, $o_i \sim o_k$
- Continuity
  -- If $o_i \quad o_j \quad o_k$, then there exists p such that $o_j \sim [p,o_i;\ (1\text{-}p),o_k]$
- Monotonicity
  -- If $o_i \quad o_j$, then, if p1 > p2, $[p1,o_i;\ (1\text{-}p1),o_j] \quad [p2,o_i;\ (1\text{-}p2),o_j]$
- Decomposability
  -- any complicated compound lottery can be converted into a simple lottery.

See Luce and Raiffa, "Games and Decisions," for more details!

The Main Result (building a utility function)

- If the axioms are met, then there exists a *linear* utility function (u(L)) over the set of probabilistic outcomes (i.e., lotteries L) arising from a finite set of outcomes.
- That is, there is a consistent way to compare the utility (value) of lotteries, allowing us to order them.
- Specifically, if $o_1 \quad o_2 \quad o_3 \ldots \quad o_n$; then
  - $u(o_1) = 1$.
  - $u(o_n) = 0$.
  - And $u(o_i)$ is gotten by indifference to o_i with certainty and the lottery $[p, o_1; (1-p), o_n]$ --which can be assessed. Then $u(o_i) = p$.

The Utility of a Lottery

- If L= $[p_1,o_1;\ p_2,o_2\ ;\ \ldots\ ;\ p_n,o_n]$, then $U(L) = \Sigma p_i u(o_i)$

- If L1 and L2 are two lotteries, then
  L1 L2 <==> u(L1) u(L2).

Do the Example in Rockower (Starts on page 15, later on page 25)

- **20 soldiers, 12 required to hold a position.**