The Other Aggregate Attrition Method Force Ratio/Fire Power Scores

Agenda

- Intro/definitions
- The FPS approach
- Attrition/movement tables, formulas, curves
- Some example scores
- Adjusting the values based on the situation
  - Adjusting for Terrain, battle posture, etc.
- The Anti-Potential Potential method to getting scores (idea in TACWAR)

The Idea

- The idea of force ratio/fire power score (FPS) attrition models is to:
  - (1) aggregate all of the individual combatants in a unit into a single scalar measure of the unit's combat power, then
  - (2) the ratio of attacker to defender combat power is used to determine the amount of combat power destroyed (attrited) by the enemy in a battle timestep and the FEBA movement.
- Used in TACWAR, RAND's JICM, Dupuy's Quantified Judgment Model (QJM)

Some definitions

- Let S_i = “value” of one system of type i
- Let X_i = Number of systems
- Then, the combat power of a force

$$= \quad {}_{(systems)}X_i * S_i$$

- Do this for each side, get FPA, FPD
- Force Ratio = FPA/FPD
- Do this for each sector (Piston model)
- Allocate casualties, movement as a function of the force ratio for a time step
- Adjust values and repeat...

Some Comments

- In the basic setup
  - Linear value of systems
  - No explicit interactions
- Very difficult to specify a system’s value

Getting Fire Power Scores

- Military judgment (RAND)
  - See picture
- Measures of Lethality (Dupuy)
  - OLI = F(rate of fire, reliability, accuracy, number of targets per strike, range)
- Anti-Potential Potential (APP)—more later
  - Objective

E.g., RAND Fire Power Scores

- This is constantly being revised
  - See JICM Homepage

Table E.2
Proposed Scores Ranked by Weapon Score

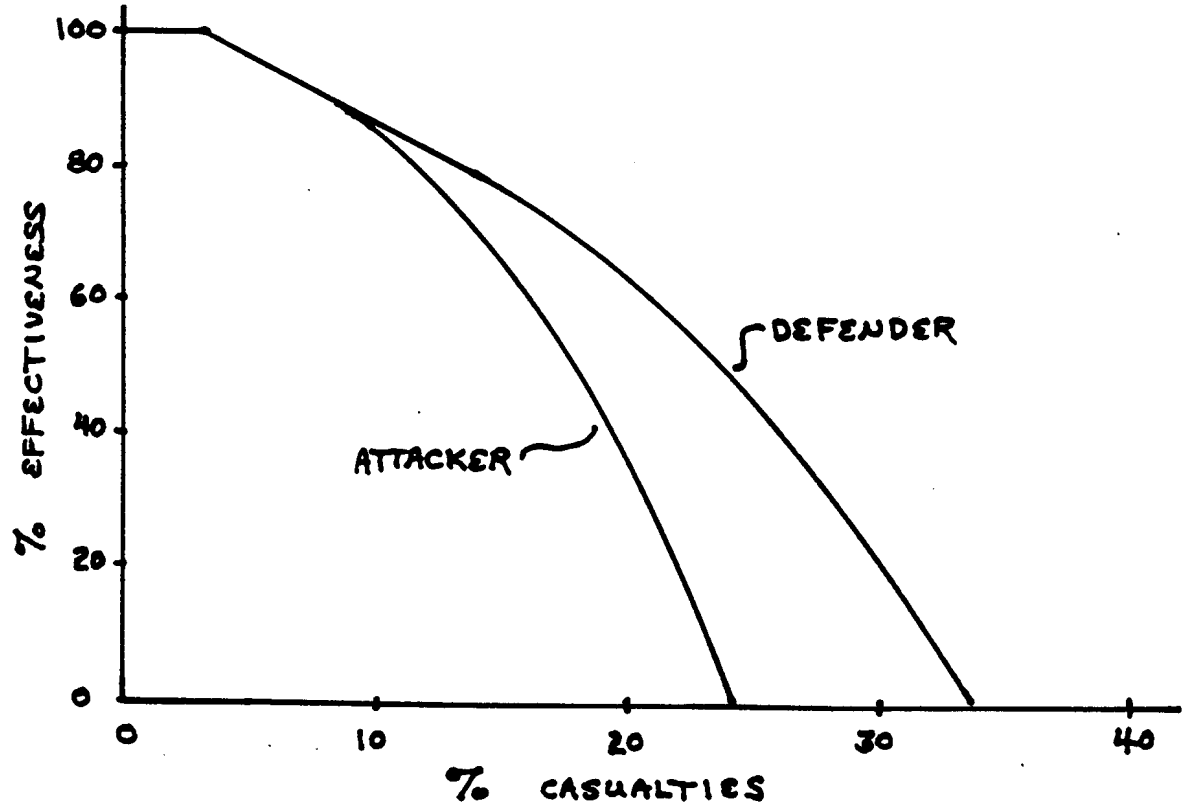
Group	Category	Score	Group	Category	Score
At Hel	AH-64	10.0	Tanks	M47	1.4
SP Arty	MLRS	10.0	ADef	Stinger	1.3
SP Arty	200+ mm MRL	7.5	APC	IFV/No AT	1.3
Tanks	M1-A1	7.5	ARV/AA	Gun/Armor	1.3
At Hel	Hind	6.5	LRAArm	Aslt/ATGN Hvy	1.2
Tanks	M1	5.5	LRAArm	Imp Tow/MP	1.2
SP Arty	152+ Hw Good	5.0	LRAArm	TOW/Mln-Veh	1.2
SP Arty	160- mm MRL Good	5.0	Mortar	SP 81 mm	1.2
SP Arty	152+ Hw Fair	4.0	Td Arty	105- mm How	1.2
At Hel	AH-1	3.5	ADef	20+ mm SP ADA	1.0
IFV/AA	M-2	3.5	ADef	57+ mm ADA	1.0
SP Arty	SP Gun	3.5	APC	APC	1.0
Tanks	M60-A3	3.5	ARV/AA	Gun/Lgt	1.0
SP Arty	160- mm MRL Fair	3.0	SRAArm	Lrg Recoil	1.0
Td Arty	122+ mm Gn/How	3.0	Tanks	T34	1.0
SP Arty	122- Hw	2.7	Td Arty	100+ mm Mortar	1.0
Td Arty	152+ mm How	2.7	ADef	SA-14	0.9
ADef	SA-13	2.5	LRAArm	Tow/Mln-MP	0.9
ARV/AA	ITV	2.5	APC	Half Trk	0.8
IFV/AA	BMP-1	2.5	LARV	Lgt Veh	0.8
Tanks	M60	2.5	LRAArm	Aslt/ATGN Lgt	0.8
Td Arty	107+ mm MRL	2.5	ADef	20-40 mm Td ADA	0.7
Td Arty	130+ mm Gun	2.5	Mortar	81 mm	0.7
At Hel	Lgt Attack	2.2	SRAArm	Sml Recoil	0.7
ADef	SA-8	1.8	ADef	SA-7	0.5
ARV/AA	ATGM	1.8	SRAArm	Dragon	0.5
Tanks	M48	1.8	ADef	AAMG	0.4
Td Arty	122-130 mm How	1.8	Mortar	60 mm	0.4
ADef	20+ mm Rad ADA	1.5	SRAArm	LAWs	0.25
ARV/AA	Lgt Tank	1.5	SRAArm	Sml LAWs	0.2
LRAArm	Imp TOW/Veh	1.5	SmArm	Small Arms	0.15
SP Arty	100+ Mortar	1.5			
Td Arty	122- mm Gun	1.5			

Adjusting Fire Power Scores

- Some examples (from TacWAR [ATLAS] and RAND)
  - Reduced effectiveness due to attrition
  - Low on logistics
  - Type of battle
  - Terrain
  - Weapons mix

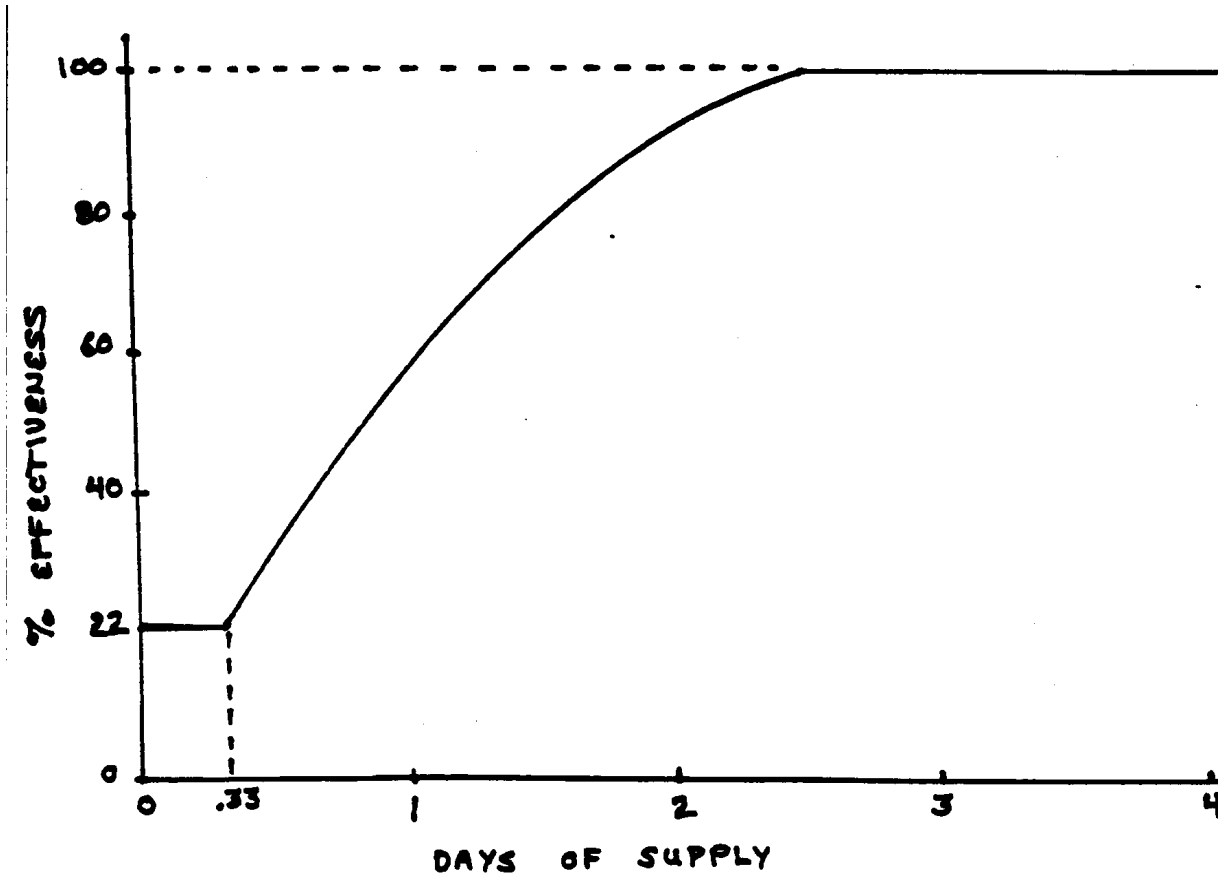
Adjusting Combat Power for…

ATLAS Percent Effectiveness Due to Casualties



Adjusting Combat Power for…

Adjust ATLAS Percent Effectiveness Due to Logistics



RAND’s Situational Force Scoring

- Adjust basic scores by factors (multiples) to account for
  - type of terrain (table lookup)
  - type of battle (table lookup)
  - Also can adjust for weapon mix nationality factors, … (again, table lookup),
  - See enclosed pictures

Example: RAND's Situational Adjustments (multipliers)

Table 3.1

Attacker Situational Category Multiplier

		Category of Weapon					
Type of Battle	Type of Terrain	att Tanks IFVs	att APCs	att LR atgms	att SR atgms	att inf	att arty
Breakthrough	Open	1.601	2.093	0.426	0.324	0.288	0.440
Breakthrough	Mixed	1.400	1.820	0.380	0.360	0.320	0.400
Breakthrough	Rough	1.260	1.638	0.787	0.972	0.864	0.810
Breakthrough	Urban	1.120	1.120	0.359	1.215	1.080	0.630
Breakthrough	Mount	1.120	1.456	0.718	1.296	1.152	0.720
Withdrawal	Open	1.380	1.794	0.532	0.405	0.360	0.550
Withdrawal	Mixed	1.200	1.560	0.475	0.450	0.400	0.500
Withdrawal	Rough	1.080	1.404	0.699	0.864	0.768	0.720
Withdrawal	Urban	0.960	0.960	0.319	1.080	0.960	0.560
Withdrawal	Mount	0.960	1.248	0.638	1.152	1.024	0.640
Delay	Open	1.150	1.495	0.638	0.486	0.432	0.660
Delay	Mixed	1.000	1.300	0.570	0.540	0.480	0.600
Delay	Rough	0.900	1.170	0.524	0.648	0.576	0.540
Delay	Urban	0.800	0.800	0.239	0.810	0.720	0.420
Delay	Mount	0.800	1.040	0.479	0.864	0.768	0.480
Hasty	Open	1.380	0.966	1.277	0.972	0.864	1.320
Hasty	Mixed	1.200	0.840	1.140	1.080	0.960	1.200
Hasty	Rough	1.080	0.756	1.049	1.296	1.152	1.080
Hasty	Urban	0.960	0.960	0.479	1.620	1.440	0.840
Hasty	Mount	0.960	0.672	0.958	1.728	1.536	0.960
Deliberate	Open	1.150	0.805	1.064	0.810	0.720	1.100
Deliberate	Mixed	1.000	0.700	0.950	0.900	0.800	1.000
Deliberate	Rough	0.900	0.630	0.874	1.080	0.960	0.900
Deliberate	Urban	0.800	0.800	0.399	1.350	1.200	0.700
Deliberate	Mount	0.800	0.560	0.798	1.440	1.280	0.800
Prepared	Open	1.093	0.765	1.011	1.013	0.684	1.045
Prepared	Mixed	0.950	0.665	0.903	1.125	0.760	0.950
Prepared	Rough	0.855	0.599	0.830	1.350	0.912	0.855
Prepared	Urban	0.760	0.760	0.379	1.688	1.140	0.665
Prepared	Mount	0.760	0.532	0.758	1.800	1.216	0.760
Fortified	Open	1.035	0.725	0.958	0.972	0.648	0.990
Fortified	Mixed	0.900	0.630	0.855	1.080	0.720	0.900
Fortified	Rough	0.810	0.567	0.787	1.296	0.864	0.810
Fortified	Urban	0.720	0.720	0.359	1.620	1.080	0.630
Fortified	Mount	0.720	0.504	0.718	1.728	1.152	0.720
Static	Open	0.920	0.920	0.560	0.450	0.450	1.320
Static	Mixed	0.800	0.800	0.500	0.500	0.500	1.200
Static	Rough	0.720	0.720	0.460	0.600	0.600	1.080
Static	Urban	0.640	0.640	0.210	0.750	0.750	0.840
Static	Mount	0.640	0.640	0.420	0.800	0.800	0.960
Meeting	Open	1.380	1.380	1.064	0.810	0.720	0.880
Meeting	Mixed	1.200	1.200	0.950	0.900	0.800	0.800
Meeting	Rough	1.080	1.080	0.874	1.080	0.960	0.720
Meeting	Urban	0.960	0.960	0.399	1.350	1.200	0.560
Meeting	Mount	0.960	0.960	0.798	1.440	1.280	0.640

Example RAND Situational Adjustments

Table 2.1

Sample Precombat Calculations in the SFS Methodology [a]

Calculation Step:	Categories of Weapons			Total
	Tanks	Inf	Arty	
1. Number of Assets (input)	300	2000	200	
2. Asset Score (multiplier from force scoring mechanism, WEI/WUV)[b]	1.0	0.05	1.0	
3. Raw Category Strength Points (row 1 times row 2)	300	100	200	600
4. Force Score Multipliers[c] (optional)	1.0	1.0	1.0	
5. Base Category Strength Points (row 3 times row 4)	300	100	200	600
6. Situational Category Multipliers (obtained from look-up table that varies by combat situation)	0.8	1.0	1.2	
7. Situational Category Strength (row 5 times row 6)	240	100	240	580
8. Shortage Category Multipliers (obtained from look-up tables that vary by combat situation)	0.8	1.0	1.0	
9. Final Category Strength (row 7 times row 8)	192	100	240	532

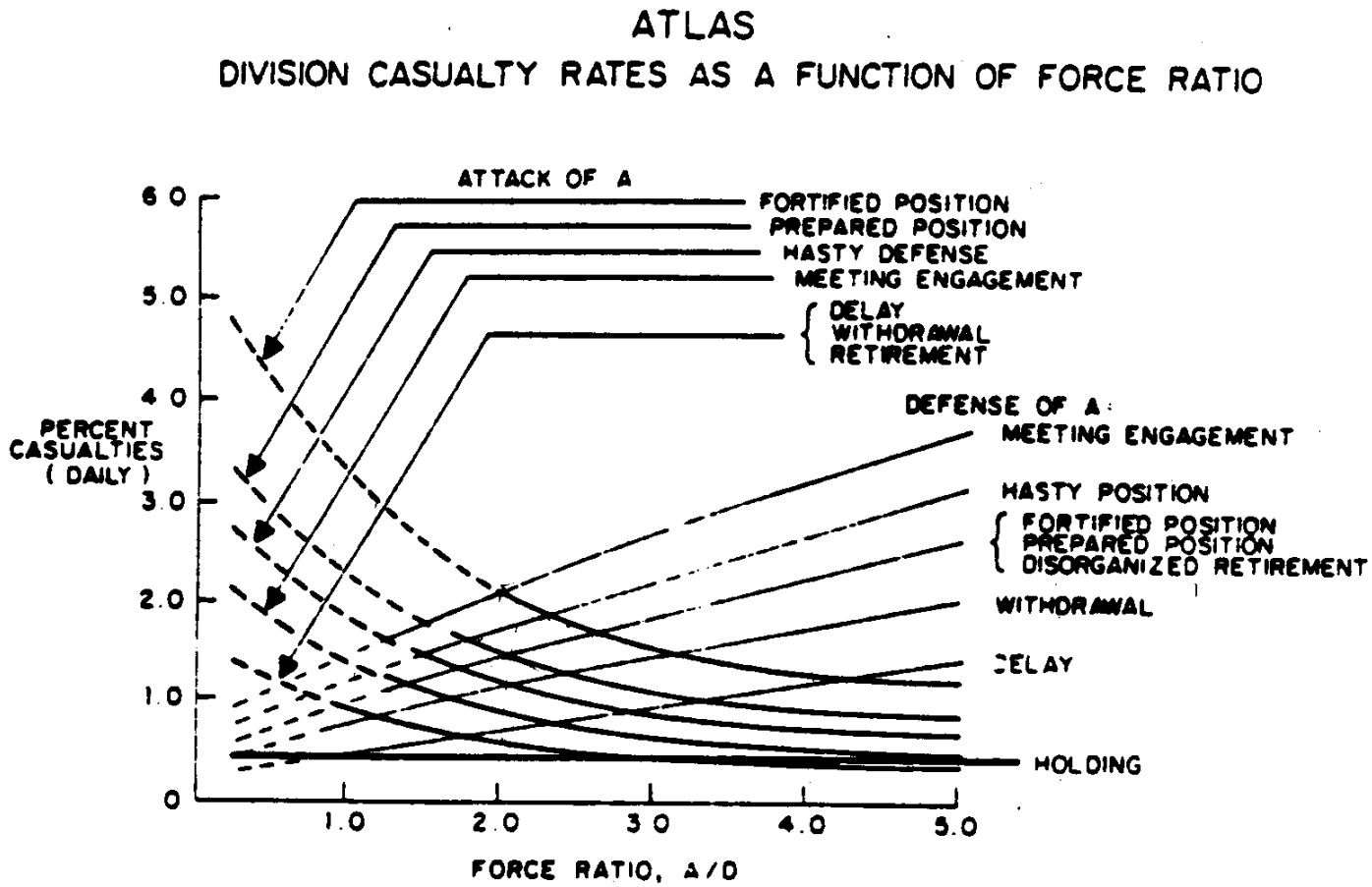
[a]Includes first two stages of SFS calculations.

[b]If the scoring mechanism includes a category weight, it is assumed to be included in this multiplier. One may wish to override the existing category weights in a scoring methodology. If so, one is then required to make sure that the category multipliers are defined with this assumption in mind.

[c]Optional force multipliers to account for the effectiveness of the unit. For example, the RSAS employs multipliers for training, nationality, and cohesiveness to determine the effectiveness of the unit before combat.

At each time step in each sector

- Calculate attrition
- In ATLAS (TACWAR)



Equations for the Curves

- *Attackers casualty rate* $= a * (x / y)^{-d} + b$
- *Defenders casualty rate* $= b * (y / x)^{-e} + a$
- *Where:* x = *attacker's combat power*
  y = *defenders combat power*
- a, b, e, a, *and* b *depend on the combat posture (make the curves)*

Engagement Type	a	b	a	b	d	e
Fortified	0.03593	0.012	0.008	0.02407	0.9	0.899021312
Prepared	0.02781	0.012	0.008	0.01919	0.87	0.899021312
Hasty	0.022384	0.015	0.01	0.016616	0.95	0.929916219
Meeting	0.02	0.02	0.011	0.011	0.89	0.89166815
Withdrawal	0.015434	0.0085	0.006	0.004566	0.93	0.879364086
Delay	0.015434	0.006	0.004	0.004566	0.93	0.778385397
Disorganized Retirement	0.015434	0.012	0.008	0.004566	0.93	0.899021312
Rout	0.015434	0.015	0.01	0.004566	0.93	0.929916219
Holding	-	-	0.009	0.009	0	0

Example

Attack of a fortified position by X on Y:
Attacker casualty rate = .03593*(A/D)^(-.9) + .02407
Defender casualty rate = .012*(D/A)^(-.899021312) + .008

Force Ratio (A/D)	Attrition Rate (A)	Attrition Rate (D)
.5	.091	.014
1	.060	.020
2	.043	.030
4	.034	.050
8	.0296	.086

RAND's Algorithm: At each time step in each sector

- Calculate attrition (some "intensity" multipliers are left out for understanding)
- In RAND's JICM (for each day)
- MFR = Force ratio adjusted for situational factors
- Defense Loss Rate (DLR) = .03*MFR^.64
- Exchange ratio´ = ER´ = 4.5*MFR^(-.57)
- Attacker Loss Rate (ALR) = (DLR*ER´)/MFR

- Let's do an example

Examples

MFR	1.000	2.000	4.000	8.000
DLR	0.030	0.047	0.073	0.114
ER'	4.500	3.031	2.042	1.375
ALR	0.135	0.071	0.037	0.020
FER (A/D)	4.500	1.516	0.510	0.172

ATLAS FEBA/FLOT Movement

ARMORED DIVISION MOVEMENT RATES (Terrain Type A, No Barriers) (In Miles per 24 Hours) FORCE RATIO (ATK / DEF)										
	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Fortified Zone	–	–	–	0.23	0.56	0.9	1.3	1.6	2.0	2.4
Prepared Position	–	–	–	0.86	2.0	3.1	4.0	4.7	5.35	6.0
Hasty Position	–	–	0.66	2.7	4.0	5.5	6.4	7.8	8.65	9.5
Meeting Engagement	–	0.0	2.9	5.6	7.3	8.9	10.3	11.6	12.4	13.6
Delaying Action	–	5.2	8.3	11.0	13.0	14.0	15.0	16.0	16.5	17.0
Orderly Retirement	0.0	13.0	16.0	17.4	18.5	18.9	19.2	19.5	19.8	20.0
Disorganized Retreat	0.0	15.0	18.1	20.0	20.8	21.5	21.9	22.2	22.4	22.6

Figure 4.3.3 – Typical ATLAS FEBA Movement Rates

RAND JICM FEBA/FLOT Movement

Then, the base value of FLOT movement rate (FMR-base) is calculated by type of battle as:

Type of Battle	FMR-Base
Breakthrough	(30.0 + 5.0 * (DLR´/ALR´))
Withdrawing	(40.0 + 6.0 * (DLR´/ALR´))
Delaying	(10.0 + 18.0 * (DLR´/ALR´))
Hasty	(0.0 + 9.0 * (DLR´/ALR´))
Deliberate	(0.0 + 12.5 * (DLR´/ALR´))
Prepared	(0.0 + 12.0 * (DLR´/ALR´))
Fortified	(-0.5 + 10.0 * (DLR´/ALR´))
Meeting	(0.0 + 5.0 * (DLR´/ALR´))
Stalemate	0

This is then combined with the intensity multiplier:

FMR = FMR-Intens * FMR-base

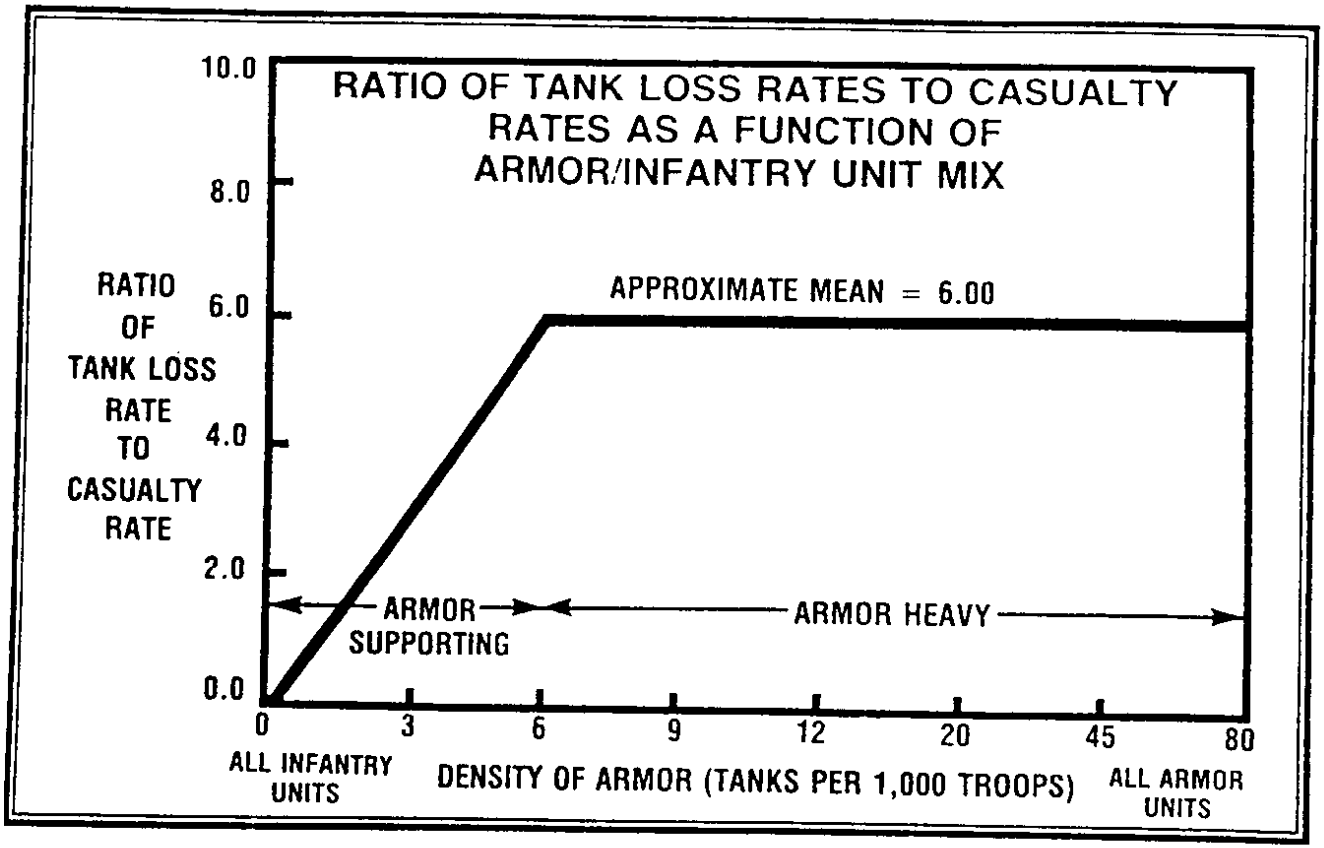
Note: DLR´ = DLR + air effects (helos + CAS)

Allocating Casualties to Systems

- Distribute equally
  - Common assumption
- Based on history
  - E.g., Dupuy
- Situational adjustments
  - E.g., RAND

Dupuy--Tank Losses as Function of Personnel Losses

Figure 59



Dupuy—Other System Losses as Function of Personnel Losses

Figure 66

Daily Loss Rates of Selected Materiel Items Normalized to a Personnel Casualty Rate of 1%

	Daily Loss Rate
Tanks	6.00 *
Artillery	0.25
Trucks	0.50
Small Arms	0.79
Mortars	1.00
Machine Guns	1.25
Radios	1.00

* Use this factor if the proportion of tanks per 1,000 troops is 6 or greater. If the proportion is less than 6, the tank loss rate factor will be the same as the number of tanks per 1,000 troops.

Dupuy has many others, e.g., annual Helicopter loss rate in Vietnam ranged from 2.7 to 7.2 times annual personnel casualty rate—see Dupuy (1995), Attrition, Nova Publications

Let's do an Example

- Indices: 1=Infantry, 2=Tanks, 3=Arty
- $X_1 = 10000$, $X_2 = 100$, $X_3 = 50$
- $S_1 = 1$, $S_2 = 10$, $S_3 = 20$
- Combat power = 12000
- Suppose, we lose 3% CP (in a time step)
  - I.e., .03*12000 = 360
- Let Y = % manpower lost
- $360 = \Sigma_{systems} S_i * m_i * Y * X_i$, where m_i is the multiplier of the loss rate relative to the base (infantry)
- Thus, 360 = 1*1*Y*10000 + 10*6*Y*100 + 20*.25*Y*50
- Solve for Y, get Y = .022154

Example continued

- Men lost = .022154*10000 = 221.54
- Tank lost = .022154*6*100 = 13.29
- Arty lost = .022154*.25*50 = .28

- Combat Power Lost = 221.54*1 + 13.29*10 + .28*20
  = 360

- Repeat in following days

RAND Losses Allocation

Table 6.1

Category Loss Multipliers

Primary Assault Weapon	Type of Battle	Attacker			Defender		
		Armor	Infty	Arty[a]	Armor	Infty	Arty[a]
Armor	Assault	1.5	1.0	0.7 * CB	1.2	1.0	0.7 * CB
Infantry	Assault	0.5	2.0	0.7 * CB	0.5	2.0	0.7 * CB
Armor	Meeting	1.5	1.0	0.7 * CB	1.5	1.0	0.7 * CB
Infantry	Meeting	0.5	2.0	0.7 * CB	0.5	2.0	0.7 * CB
Armor	Stalemate	1.5	1.0	2.0 * CB	1.5	1.0	2.0 * CB
Infantry	Stalemate	0.5	1.0	2.0 * CB	0.5	1.0	2.0 * CB
—	Withdrawal	2.0	1.0	0.2 * CB	1.2	1.0	0.4 * CB + 0.4
—	Breakthrough	2.0	1.0	0.2 * CB	1.2	1.0	0.4 * CB + 0.4
—	Delay	1.5	1.0	0.7 * CB	1.5	1.0	0.5 * CB

[a]"CB" is the opponent's counterbattery capability as discussed in Section 4.

An Objective Approach to Assigning System Fire Power Scores: Anti-Potential Potential (APP)

- **The Basic Idea**
  - The value (score) of a weapon system is directly proportional to the rate at which it destroys the value of opposing enemy weapon systems.
  - Thus the value of a system depends on its kill rates and on the value of the enemy systems it kills.

Some Notation

- Two opposing forces (X and Y) made up of heterogeneous weapon systems.
- Let X_i = the number of weapons of type i in the X force for i = 1,2,…,m,
  with $\underline{X}^t = \{X_1, \ldots, X_m\}$
  Y_j =the number of weapons of type j in the Y force for j = 1,2,…,n,
  with $\underline{Y}^t = \{Y_1, \ldots, Y_n\}$
- Define the weapon values (or scores to be determined) as:
- SX_i = the value of one type i weapon in the X force, with
  $S\underline{X}^t = \{SX_1, \ldots, SX_m\}$
- SY_j = the value of one type j weapon in the Y force, with
  $S\underline{Y}^t = \{SY_1, \ldots, SY_n\}$

More Notation

- Need to define kill rates
  - Let K_{ij} = the rate at which one X_i system kills Y_j systems (with K the $m \times n$ matrix)
    L_{ji} = the rate at which one Y_j system kills X_i systems (with L the $n \times m$ matrix)
- Kill rate values are usually obtained from the killer-victim scoreboards from high-resolution simulations

Applying the APP principle

- The value for a system of type i in the X force is given by totaling the values of the enemy system that it kills:

$$CX \times SX_i = \sum_j K_{ij} \times SY_j$$

- Similarly for the Y force:
- $$CY \times SY_j = \sum_i L_{ji} \times SX_i$$
- CX and CY are proportionality constants (more later)
- Total of m+n equations and m+n unknown values

Put in Matrix Form

(1) $CX \times \underline{SX} = K \times \underline{SY}$

(2) $CY \times \underline{SY} = L \times \underline{SX}$

- Substitute (2) in (1),
- get $CX \times CY \times \underline{SX} = (K \times L)_{(m \times m)} \times \underline{SX}$
- Let $\lambda = CX \times CY$,
- Facts:
  - (1) a non-negative, largest eigenvalue λ (the same for both equation systems), and
  - (2) non-negative eigenvectors $\underline{SX}$ and $\underline{SY}$ (unique up to a scale factor) that satisfy the equations.

In Practice

- Solve for SX
  - Scale by taking a base weapon system (say M1A1 tank) and set equal to 1. This gives all other SX values.
- Given SX, solve for SY directly
- We will let the computer do the calculations

- This gives the APP scores

Let’s Do an Example

- In S-plus/see attached