Non-monotonicity, Chaos and Combat Models

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Overview

- Introduction of basic model concepts
- Description of model
- Non-monotonicity in combat models
- Chaos in the underlying model
- Connecting chaos with non-monotonicities
- Summary

Expectations of Computerized Combat Models

- Somewhat limited in scope of analysis
  - i.e., measurement of leadership, “fog of war”, reaction of enemy forces
- Relatively predictive
  - Indicating when one system or configuration is better than another
- Monotonic
  - Adding more capabilities to one side will lead to at least as favorable a combat outcome on that side

Non-monotonic Behavior

- Basic Causes
  - Stochastic variables
  - Nonlinearities
- Treatments
  - "Fix" a specific, observed non-monotonicity until it disappears
  - Ignore it as you would an anomaly

Characteristics of Chaos

- Unpredictable, random-looking behavior
- Extreme sensitivity to initial conditions
- Not a product of randomness
- Nonlinear with feedback

Non-monotonicity and Chaos in the Combat Model

- Challenges to investigating
  - Result of nonlinearities associated with potential chaos
  - Must carefully choose definition of chaos appropriate to the model
  - Chaos is a long term behavior, models are normally run for a relatively small number of steps
  - Clarify relationship between chaos and perceived misbehavior

The Model

- Discussion requires a simple, yet reasonable, model
  - Initial troop strengths
  - Attrition coefficients
  - Reinforcement thresholds
  - Time steps
  - Stopping criteria

Monotonic Reaction

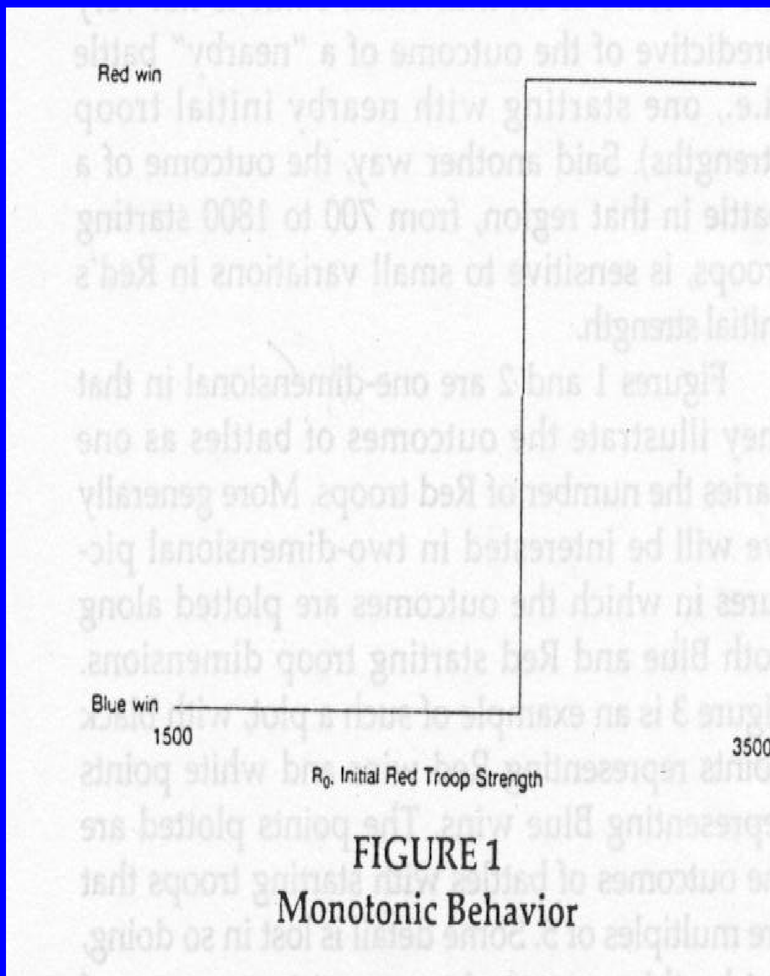

FIGURE 1
Monotonic Behavior

- Blue forces fixed at 839
- Red forces ranging from 1500 to 3500

Non-monotonic Reaction

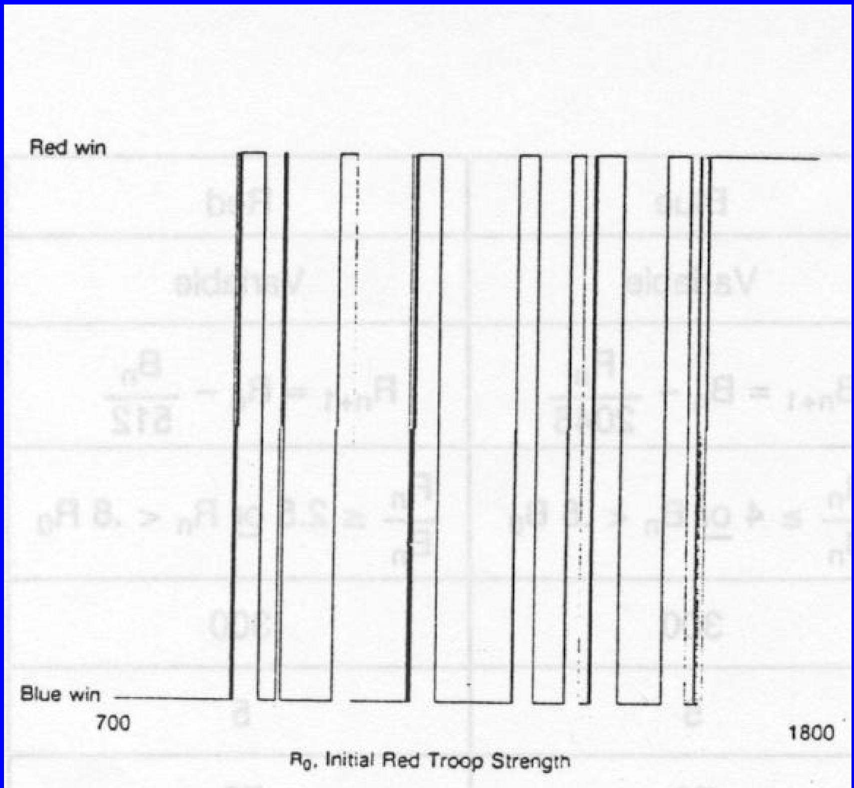

FIGURE 2

- Blue forces fixed at 500
- Red forces ranging from 700 to 1800

Causes of Non-monotonicity

- Causes of non-monotonicity
  - Time step granularity problems
  - Delayed feedback effects
  - Round-off/Precision effects
  - Random variable effects
  - Smoothing and Time averaging effects
  - Mathematical Chaos

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Eliminating Non-monotonicity

- Time step granularity
  - Model uses discrete approximation to differential equation
- Fix: Decrease time step

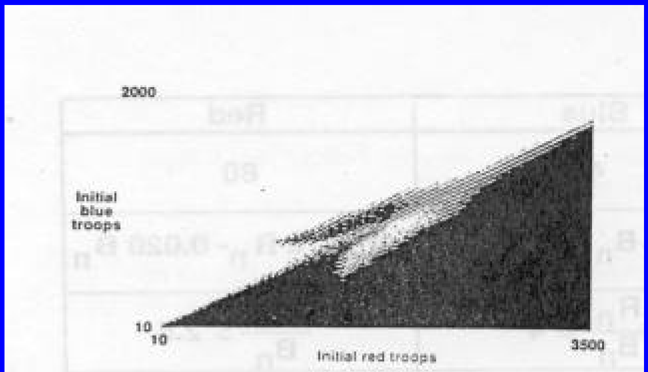

FIGURE 4
Inappropriate Time Step Size

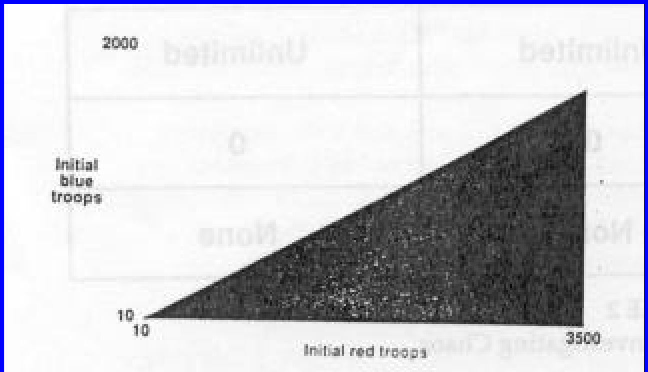

FIGURE 5
Appropriate Time Step Size

Eliminating Non-monotonicity

- Delayed feedback effects
  - non-linear feedback loop causes instabilities
- Fix: Set reinforcement delay to zero
  - use scripted reinforcements

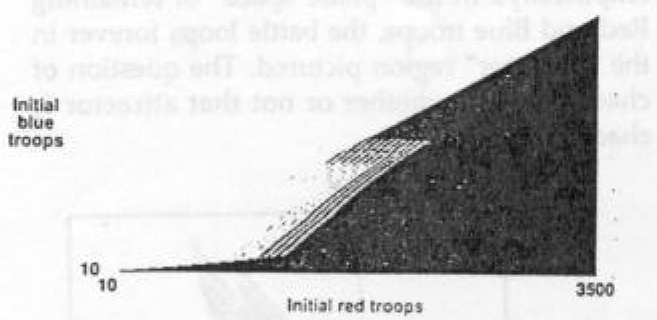

FIGURE 6
No Reinforcement Delays

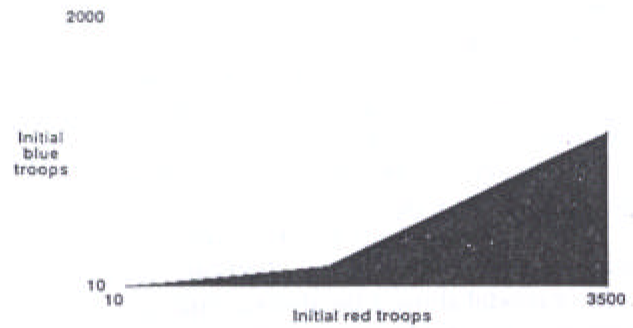

FIGURE 7
Scripted Reinforcements

Eliminating Non-monotonicity

- Round-off/precision effects
  - floating point decimal errors
  - irrational numbers
- Fix: Attrition coefficients chosen to be powers of 2 to minimize problems

Non-monotonicity review

- Non-monotonic behavior is bad
- Eliminate behavior when possible
  - Use small time steps
    - inherent in attrition and delay coefficients
  - Use time scripted reinforcements
  - Choose input numbers wisely
- Minimize non-monotonic region

Proving There is Chaos in the Underlying Model

- **Chaos Defined:**
  - **"**There is still no generally accepted definition of chaos"(p 43).

- For a set V. I*f*: V→V is chaotic on V if:
  - I*f* has **sensitive dependence on initial conditions** (making it unpredictable).
  - I*f* is **topologically transitive** (a point in a subspace of V always maps or "loops" back to that same subspace, precluding the decomposition of V into two distinct subsets).
  - Periodic points of I*f* are **dense in V** (resulting in a certain degree of "regularity" in the function)

Proving There is Chaos in the Underlying Model:The "Infinite Model"

- Removal of artificial stopping conditions isolates the dynamics of the model itself by allowing the asymptotic behavior of the model to be studied (i.e. the behavior of the underlying mathematics as time $\rightarrow \infty$)

The "Infinite Model"

	Blue	Red
Initial troop strength	40	80
Combat attrition calculation	$B_{n+1} = B_n - 0.005\, R_n$	$R_{n+1} = R_n - 0.020\, B_n$
Reinforcement thresholds	$\frac{R_n}{B_n} \geq 4$	$\frac{R_n}{B_n} \leq 2.5$
Reinforcement block size	10	10
Maximum allowable reinforcement blocks	Unlimited	Unlimited
Reinforcement delay (time steps)	0	0
Withdrawal thresholds	None	None

TABLE 2
Modified Model for Investigating Chaos

Result: The Asymptotic Behavior of the Model

- Pretty obvious interpretation . . . any questions?
- Remaining Red and Blue Forces loop forever in the "attractor" region
- But is the attractor region chaotic?

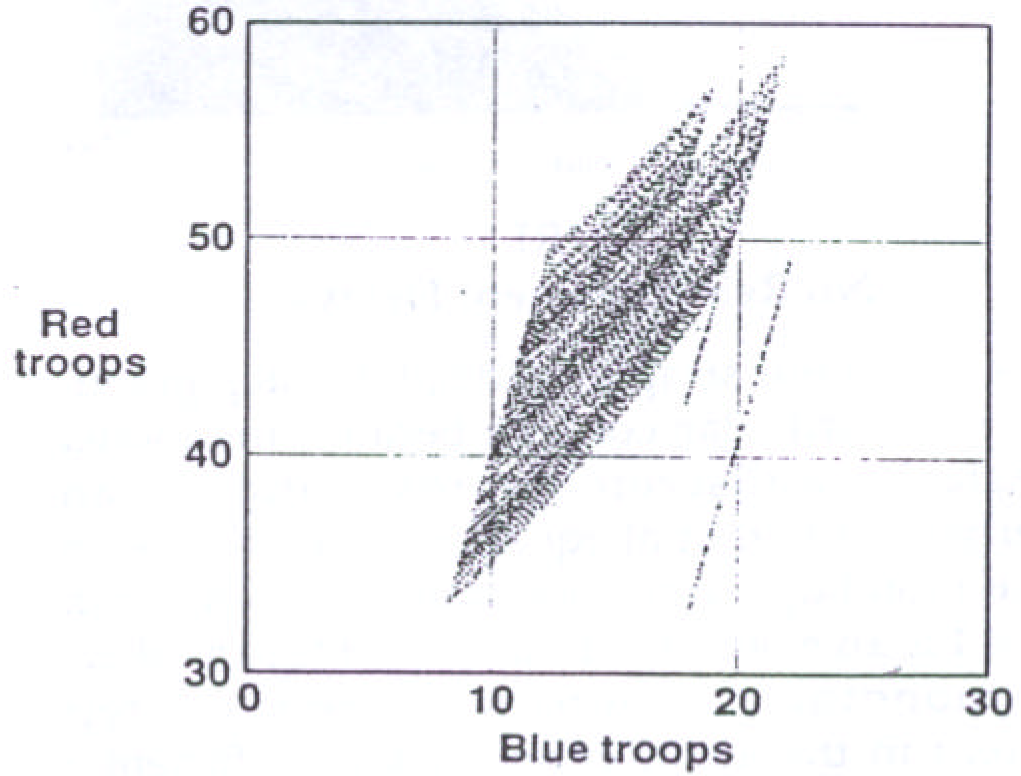

FIGURE 8
Phase Space Attractor

Proving There is Chaos in the Underlying Model: Evaluation

Hey, I thought there was no accepted definition of chaos

- The "Infinite Model" doesn't "strictly meet" the above "chaos criteria," but shows . . .
- Sensitive dependence on I.C.'s
- Topological Transitivity
- An Infinite Number of Periodic Points (that's pretty dense)
- Invariant density (with gaps)

Connecting Chaos with Non-Monotonicities

- Chaos is a Long Run Effect

  -Must “take care” is comparing infinite model with finite combat model.

- Disappearance of non-monotonicities from finite model when chaos removed from infinite model ⟶ Strong Evidence of link . . . Recall:

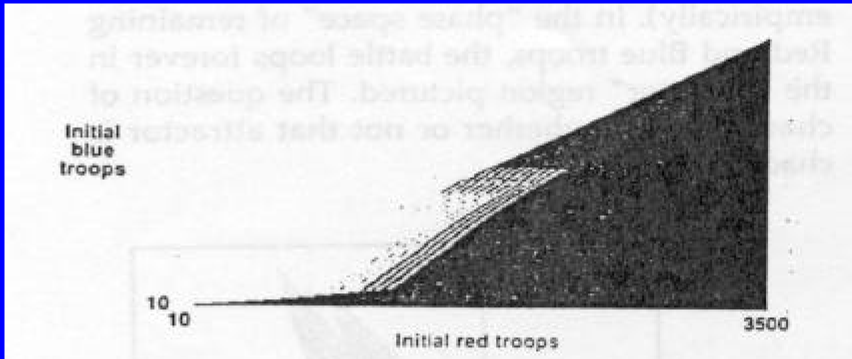

FIGURE 6
No Reinforcement Delays

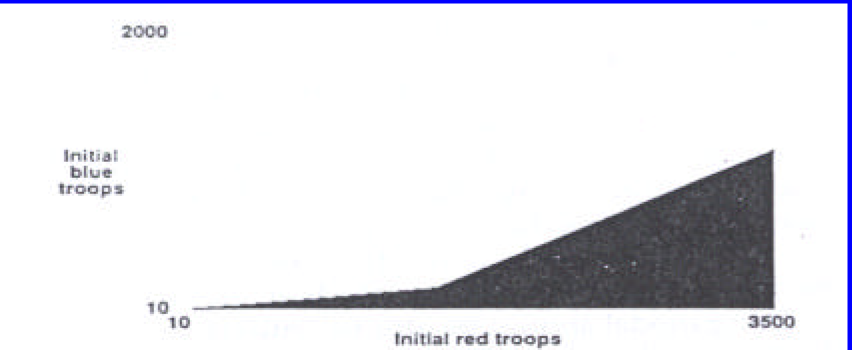

FIGURE 7
Scripted Reinforcements

Connecting Chaos with Non-Monotonicities

- Big Problem: the post-battle assignment of final states (e.g. “win” or “lose”) based upon force levels at stop time.
- Non-monotonicity = unexpected reversal of fortune
- Specific challenge: show that the reinforcement decision is causing the problem

Connecting Chaos with Non-Monotonicities: Adjusting the Model One More Time

- Only two differences between the finite and infinite models:
  - Stopping criteria
  - Possible number of reinforcements
- Remove stopping criteria from finite model
- Restrict reinforcement levels to ensure finite battles
- Any non-monotonicities must be the result of the reinforcement heuristic

Connecting Chaos with Non-Monotonicities: The Results

- There were still non-monotonicities
- The nonlinearities from the reinforcement heuristic produced unexpected outcomes
- Therefore chaos leads to non-monotonic behavior in the model
- QED (more or less)

Implications for Larger Models

- More state dependent thresholds
- Ambiguity region

Attrition Only Submodel

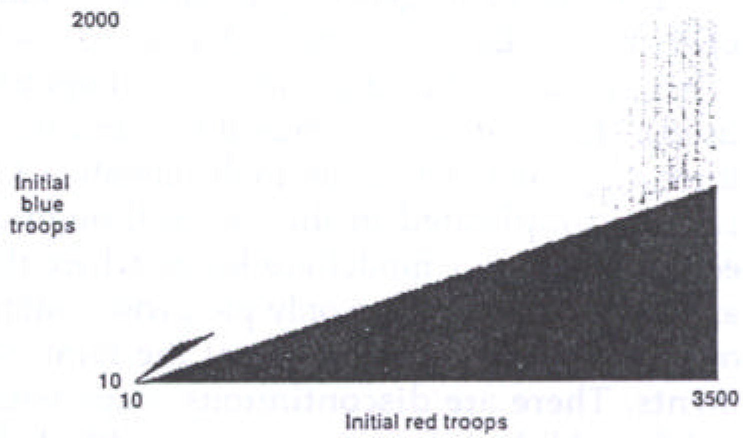

FIGURE 9
Simple Model: Attrition Thresholds Only

Force Ratio Only Submodel

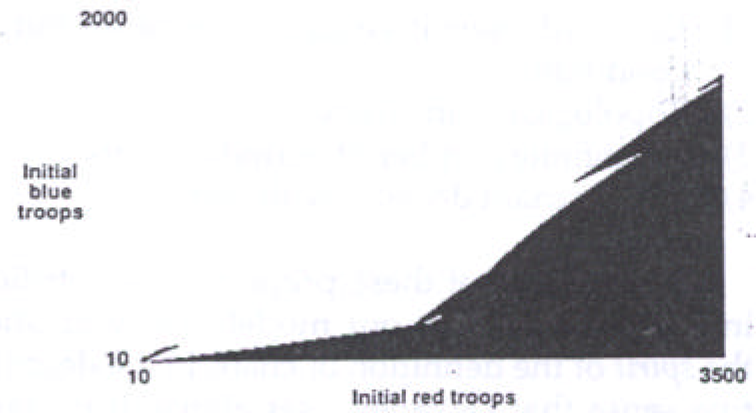

FIGURE 10
Simple Model: Force Ratio Thresholds Only

"Wash Out" Counterexample

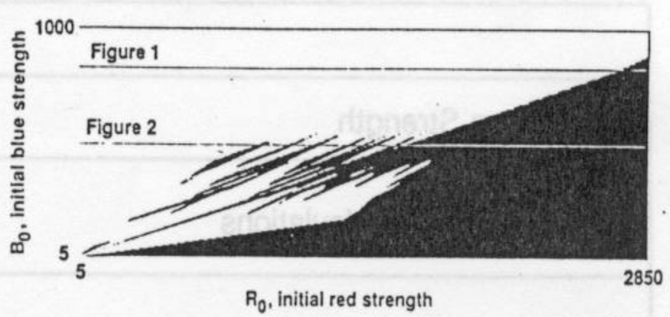

FIGURE 3
Simple Model Outcomes in Two Dimensions

Ambiguity Region

- Shrinking non-monotonic area hypothesis
- Can describe region analytically
- Optimal strategy for each submodel
  - Instant unilateral reinforcements
  - Possible non-monotonicity demarcation line

Ambiguity Region

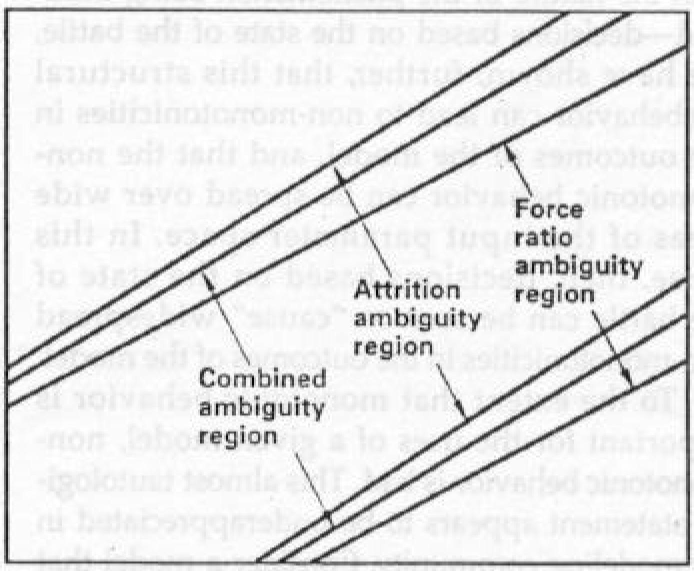

FIGURE 12
Combining Ambiguity Regions

- Combine demarcation lines to form Ambiguity Region
- Is **NOT** the union or intersection of the submodels

Summary

- Model with one decision variable satisfies mathematical conditions of chaos
- Chaotic dynamics of system cause

  non-monotonicities
- Appear even in an exactly solved system
- Exists over a wide range of initial conditions

Why Are Non-Monotonicities Bad?

- Consider a model comparing two alternatives
- Non-monotonic model reflects contributions of alternatives as well as underlying chaos
- Statistics example

How to Insure Valid Model Results

- Remove decisions that cause non-monotonicities
- Limit area of interest to monotonic region

Non-Monotonicity, Chaos and Combat Models

- What defines non-monotonic behavior?

Non-Monotonicity, Chaos and Combat Models

- What defines non-monotonic behavior?
  - "Any unexpected reversal in outcome associated with a given change in inputs" (p 43).

Non-Monotonicity, Chaos and Combat Models

- Name two causes of non-monotonicity.

Non-Monotonicity, Chaos and Combat Models

- Name two causes of non-monotonicity.
  - Time step granularity problems
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Non-Monotonicity, Chaos and Combat Models

- How do you insure valid model results?

Non-Monotonicity, Chaos and Combat Models

- How do you insure valid model results?
  - Remove decision variables that cause non-monotonic behavior
  - Limit area of interest to monotonic region

Non-Monotonicity, Chaos and Combat Models

QUESTIONS?