Salvo Model of Warships in Missile Combat

Presented by:

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Warship Missile Combat Salvo Model Topics

- Balanced Warship Design
- Design Criteria for Exploratory Analysis
- Salvo Model Assumptions
- Instability of Modern Forces in Salvo Warfare
- Terms
- Conclusions

Balanced Warship Design

- Balance factors:
  - Offensive Power
  - Defensive Power
  - Staying Power

- Dilemmas:
  - Damage Variance is High in Warships
  - Military Value of Staying Power?

Balanced Warship Design

- Staying Power:
  - Downplayed after advent of nuclear weapons
  - Did not return after conventional power became cool again
  - Conflicts up to the present do not involve significant conventional naval threats

Balanced Warship Design

- Analysis of major ship attributes is required
  - Use Force on Force analysis
  - Missile warfare needs modelling processes that differ from historic models
  - Staying power emphasised as a factor in ship design

Design Criteria

- Best Measure of Warship Productivity?
  - Quantity of accurately delivered ordnance over a ships combat life
- Best Measure of Naval Force Productivity?
  - Quantity of accurately delivered lethality over a group of ships combat life

Design Criteria

- Other Influential Values:
  - Ordnance Hit Probability
  - Timing of fire from different ships in a force
  - Endurance of fire from different ships in a force
  - Distribution of fire among targets
  - Ambush, or other tactical advantages

Basic Salvo Equations

Force-on-force equations for combat achieved by a single salvo at any time step

? B = (a A – b_3B) / b_1

? A = (ßB – a_3A) / a_1

Fighting Strength

- Military worth of a force
- Who wins a salvo exchange?

A wins if

$$a_1 a A^2 - a_1 A b_3 B > b_1 ßB^2 - b_1 B a_3 A$$

Otherwise B wins the salvo exchange.

Model-Based Conclusions 1

- Fraction of force that can be put out of action by a salvo
  ? B / B = (aA – b_3B) / b_1B
- Fractional Exchange Ratio (FER)
  FER = (? B / B) / (? A / A)
  Comparative fighting strength of A and B
- FER > 1
  => A has forces remaining when B is out of action

Model-Based Conclusions 2

- Excessive offensive and defensive power have a significant effect on results. The FER must be used with caution when overkill exists.
- When A has twice the striking power, twice the defensive power and twice the staying power of B, B can still achieve parity (FER = 1) if its force is twice as numerous as A.

Discussion

Number of units	A = 2	B = 6
Staying power	$a_1 = 2$	$b_1 = 1$
Defensive power	$a_3 = 16$	$b_3 = 1$
Striking power	a = 24	ß = 6

FER = (? B / B) / (? A / A) = 7.0 => A wins

Fighting strength

$$a_1 a A^2 - a_1 A b_3 B > b_1 ß B^2 - b_1 B a_3 A$$

$$\Rightarrow \quad 192 - 24 > 216 - 192$$

⇒ A wins the salvo exchange!

Discussion 2

Fraction of each force that can be put out of action

? B / B = 7.0 — A can put B out of action 7 times with one salvo

? A / A = 1.0 — B can put A out of action with one salvo

<u>Result:</u>

Despite great advantage in offensive and defensive power, A cannot become involved in an exchange.

B can take out a force of far more fighting strength, but B's is a suicidal task.

Salvo Equation Embellishment

Add terms for:

- Scouting
- Defensive Readiness
- Soft Kill
- Skill/Training

-Instead of 0 or 1, for offense and defense, use multipliers to enrich the analytical potential of the model.

-This enrichment also complicates and confuses the basic understanding of the interrelationships.

Assumptions

1. Scouting Effectiveness: $\blacklozenge_A$ or $\blacklozenge_B$ have values between 0 and 1, measuring Striking Power. It diminishes due to less than perfect targeting and distribution of fire against the target force.

2. Defender Alertness and Readiness: $\underline{\Omega}_A$ or $\underline{\Omega}_B$, also takes values between 0 and 1. Measures counter-fire ability the force. Diminishes due to less than perfect readiness or fire-control designation.

Assumptions (cont)

3. Seduction Chaff : a_4 or b_4 causes the accurate missile shots to miss. Assumed that the probability is the same for each missile, and the probability does not change with the number of defenders.

**Stealth, and avoidance by maneuver is treated the same mathematically.

Assumptions (cont)

4. Distraction Chaff: ❒$_A$ or ❒$_B$, Draws off shots prior to counter-fire. Reduces the number of good shots to be destroyed by counter-fire. A given probability each enemy shot will be distracted. Applied to ♌B and ♋A respectively.

5. Training, Skill and Motivation: ♦$_A$ and ♦$_B$ measure the degree to which the unit (firing or target) fails to achieve its potential.

Force-on-Force Equations

♋’ = ♦$_A$ ◆$_A$ ❒$_B$ ♋ is the fighting power in hits of an attacking unit of side A, modified for Scouting, Training and defender B’s distraction chaff effectiveness.

♌’ = ♦$_B$ ◆$_B$ ❒$_A$ ♌ - same for side B

b_3’ = ♎$_B$ ◆$_B$ b_3 is the hits denied to side A by side B’s counter-fire, with alertness and training efficiencies factored in.

a_3’ = ♎$_A$ ◆$_A$ a_3 – same for side A.

Force-on-Force Equations(cont)

👎👌💾(♋❍✌📪b_3´👌◑b_4/b_1

👎✌💾☎♌❍👌📪a_3❍A)a_4/a_1

<u>CONCLUSIONS:</u>

-Striking Power (♋❍⦿♌❍) and Defensive Power (a_3❍/b_3❍) depend on good scouting. The number of participating forces (A and B) depends on leadership and tactics.

-Force Staying Power (a_1A/b_1B) is in the design and is not dependent on scouting or tactical concentration.

Conclusions (cont)

2. For analysis, the effect of scouting can be reduced to the four multipliers ♦$_A$, ♦$_B$, ♎$_A$ and ♎$_B$. The factors diminish each sides striking power and counter-fire.

3. Training deficiencies can be incorporated in ♦ for their respective sides.

Discussion

-Scouting's principal failure mathematically is combining the scouting and shooting process into a system for evaluation.

-We know where to put scouting into the equations, particularly degrading defensive power and that it is vital in modern cruise missile warfare.

-Making up for a lack of scouting is to increase the staying power or the number of units.

Conclusions

-Ship attributes are all important, none can be left out of a design. The Salvo equation allows for the study of the interrelationships between the attributes.

-Enemy Attributes and FER: The numbers and quality of the enemy are an important part of the outcome of a battle. FER allows for an abstract way to compare warship attributes in the absence of knowledge about an enemy which is unknown during the building of a ship.

Conclusions(cont)

-Command and Control: Does not show up in the equations, however is imbedded into other variables in the equation.

-Staying Power: Built into the design of the ship, has no relation to tactics during the battle.

-Numbers: Linear Law applies, if we have twice as many ships, all of the enemies ships must have twice as good Staying Power, Striking Power, and Defensive Power.

Conclusions(cont)

-Quantity vs Quality: Fleets weak in Staying Power cause unstable situations, they are risk averse because one missile can be devastating.

-Tactical Instability: As the ratio of Combat Power to Staying Power increases, Instability also increases. There is more chance for the inferior side to win or do great damage.

-Scouting: Can have long range weapons, but they are ineffective without scouting. (e.g. TASM)

Conclusions(cont)

-Design Goal: Maximum fighting strength is a design goal. There is no consistent favor for Striking Power, Staying Power or Defensive Power over the others.

-Staying Power: No USW involved in the equations. The value of Staying Power is understated for USW. Littoral environments will decrease the defensive effectiveness due to the short response times.

Possible Exam-Question

Compute the outcome of a single salvo exchange:

- Compute ? A and ? B
- Compute FER
- Compute the salvo effects in an exchange

Possible Exam-Question

What is the most important aspect of modern salvo warfare?

- Scouting

Possible Exam-Question

What law does the Salvo Equation follow?

- Linear Law

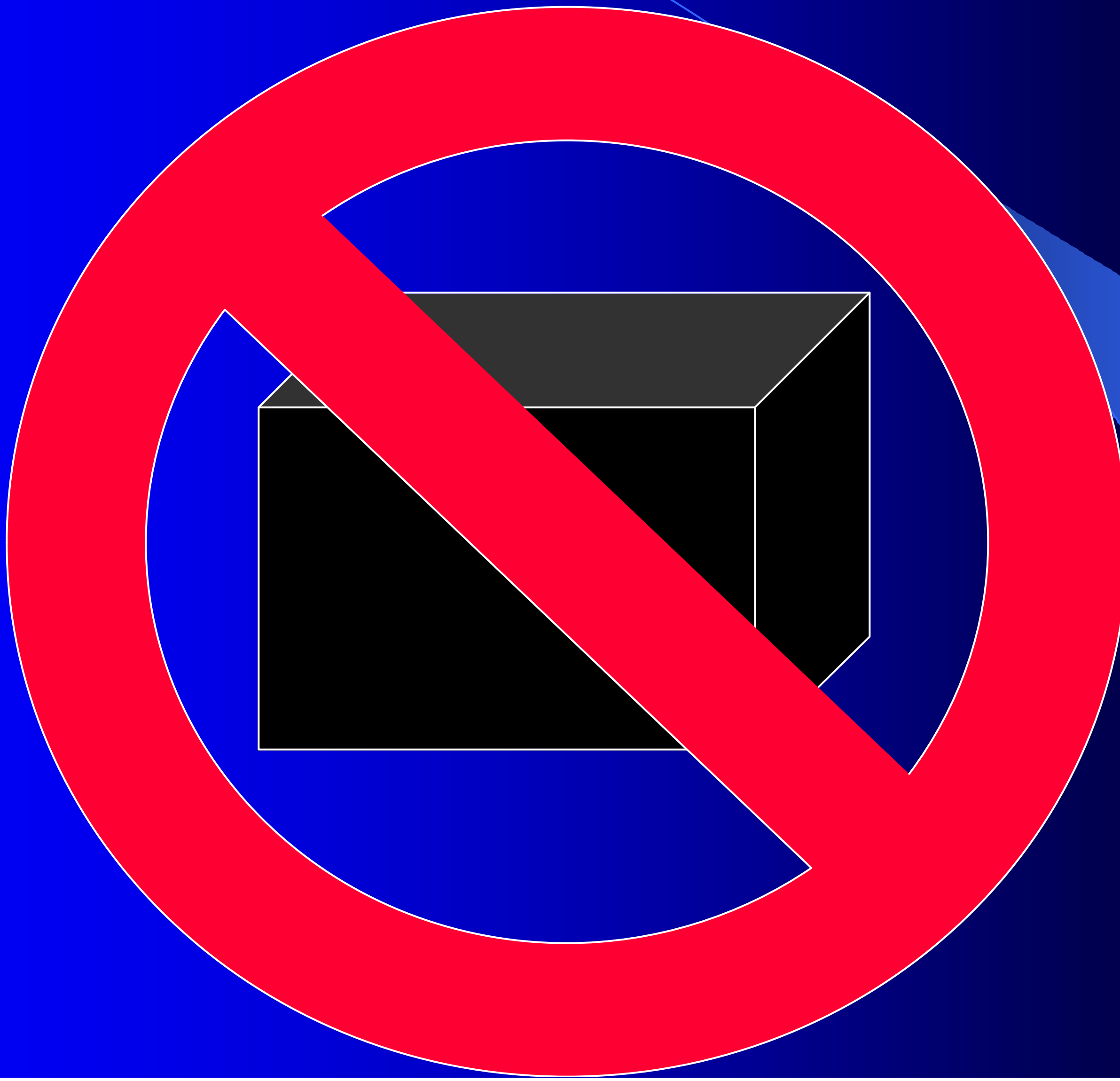
Variables

A	= # units in force A
B	= # units in force B
a	= # well-aimed missiles fired by each unit A
ß	= # well-aimed missiles fired by each unit B
a_1	= # hits by B’s missiles needed to put one A out of action
b_1	= # hits by A’s missiles needed to put one B out of action
a_3	= # well-aimed missiles destroyed by each A
b_3	= # well-aimed missiles destroyed by each B
? A	= # units in force A out of action from B’s salvo
? B	= # units in force B out of action from A’s salvo

Hughes Salvo Model Follow-up

- *"Modeling difficulty increases exponentially and explainability goes to zero as the number of objects increases"—Kirk Yost*
- *"[A] large-scale computer model may…restrict rather than extend thought stems simply from the size and complexity"—The Committee*
- *"A well-schooled man is one who searches for the degree of precision in each kind of study which the nature of the subject at hand admits"—Aristotle*
- *All models are wrong, but some are useful—George Box*
- *"Too many analysts…regard OA as the art and science of building realistic models…with only passing regard for the utility and efficiency of their representations in helping decision-makers"—Wayne Hughes*

No Black Box Models For Analysis



The Base Hughes Salvo Model

$$\Delta B = \frac{aA - b_3 B}{b_1}$$

$$\Delta A = \frac{bB - a_3 A}{a_1}$$

- A, B = Number of units in the forces
- ΔA, ΔB = Change in forces after salvo exchange
- α, β = Offensive Power of (A, B)
- a_3, b_3 = Defensive Power of (A, B)
- a_1, b_1 = Staying Power of (A, B)

Calculating Proportional Losses

$$\frac{\Delta B}{B} = \frac{aA - b_3 B}{b_1 B}$$

$$\frac{\Delta A}{A} = \frac{bB - a_3 A}{a_1 A}$$

Losses can be efficiently minimized by

(1) Increasing staying power

(2) Having more ships

Comparing Fighting Strengths

The Salvo equations are the square law on steroids!

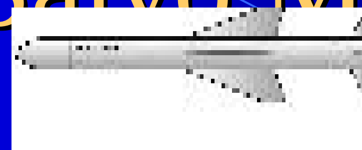
$$FER = \frac{\Delta B / B}{\Delta A / A} = \frac{(aA - b_3 B)(a_1 A)}{(bB - a_3 A)(b_1 B)}$$

Some Insights

- The advantage of numerical superiority
  - “Though an obstinate fight may be made by a small force, in the end it must be captured by the larger force”—Sun Tzu
  - “Numerical superiority is the force attribute that is consistently most advantageous”—Wayne Hughes
- Instability of modern forces in salvo warfare
  - Instability exists when staying power is weak relative to combat power (sort of like MAD)
- “Fire effectively first!”—Wayne Hughes

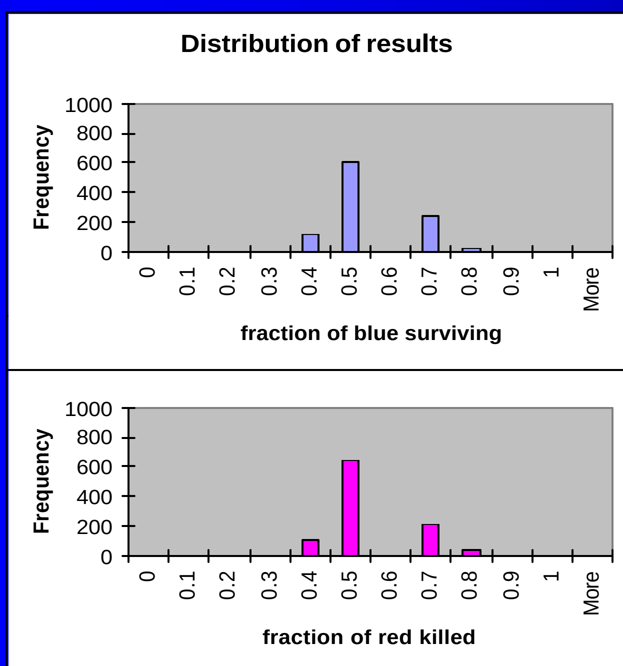
Transparent models allow for transparent arguments

Stochastic Extension to the Hughes Salvo Model



Exploratory Analysis 120 cases

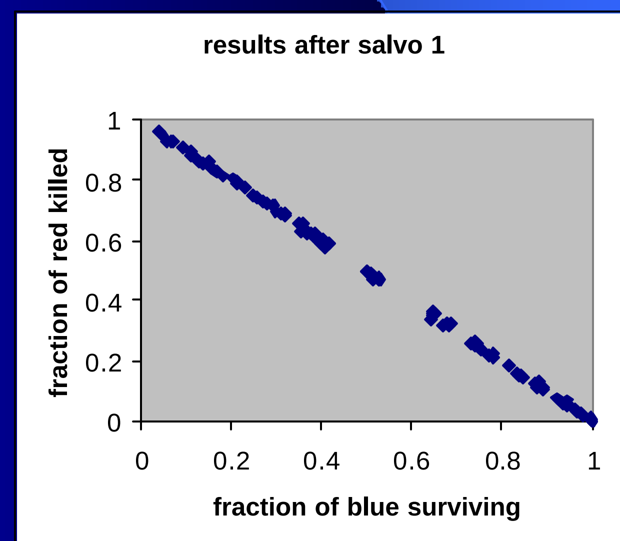
Units:	**2-6**
Striking capability:	**1-4**
Defensive capability:	**1-3**
Staying power:	**1-2**



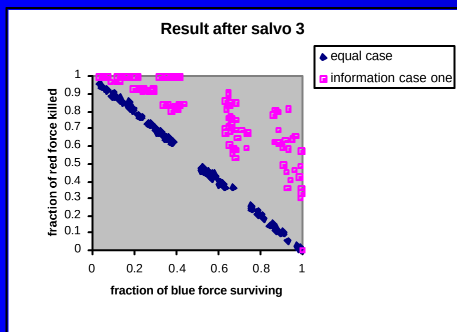
X

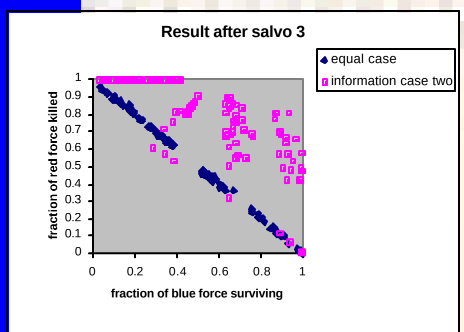
Y

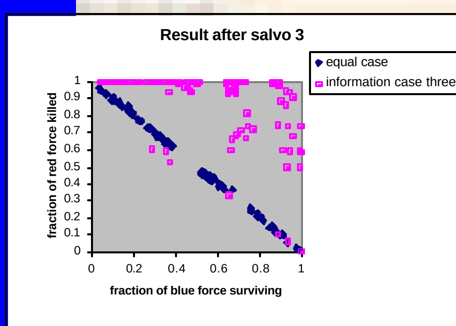
Plot of the Offensive vs. Defense measures of effectiveness



Information advantage



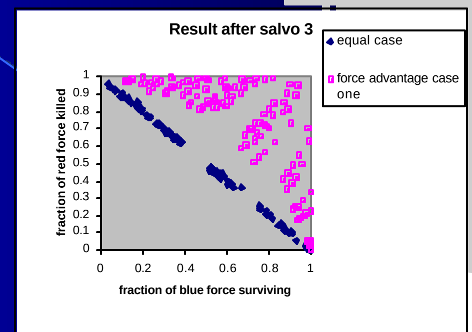




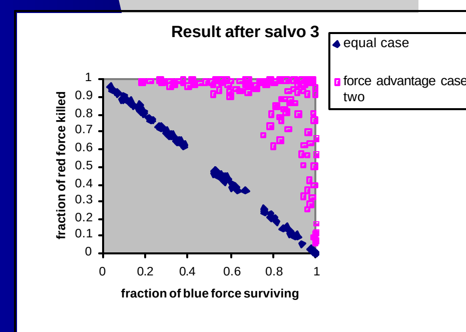
More information

Even more information

Force advantage



One unit



Two units

Future Directions

- ***At least ten theses have been based on the Salvo model***
- ***McGunnigle's model with unlike forces***
- ***Heterogeneous Salvo model (Johns, December 2000)***
  - ***A stochastic heterogeneous model and the value of information?***
- ***Other?***